COMPUTER MODELING OF NON-STATIONARY GAS QUASI-KEPLERIAN DISK

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Abstract We have considered convective instability in a plane of a disk. In this case nonaxisymmetric perturbations are unstable, and the reason of a convection is connected with a radial non-homogeneity of thermodynamic parameters in quasi-Keplerian disk. On the basis of the linear analysis of stability in WKB-approximation the borders of stability are received. The hydrodynamical model of a non-stationary gas disk in the gravitational field of dot mass is constructed. We have studied nonlinear dynamics of convective unstable perturbations because of radial non-homogeneous entropy, neglecting effects of cooling and viscosity. The opportunity of formation of spiral-cellular structure of convective perturbations with shock waves is shown here.

Keywords: hydrodynamics, accretion disk, instability, convection

1. Problem of a convection in accretion disks

Within the framework of accretion disk (AD) standard α-model (Shakura & Sunyaev) there is a number of theoretical problems and problems connected with the explanation of the observation data. The latter is concerned with the explanation of low luminosity of X-ray binary system and active galactic nucleus with black holes (see the review (Narayan 2002) and references there). One of the most investigated objects of such type is the source in the centre to Galaxy Sgr A^* (Melia & Falcke). The reduction of luminosity is provided in so-called ADAF-models (advection-dominated accretion flows) (Abramowicz et al. 1995). They are much hotter and Eddington luminosity is reached at smaller rate of accretion.

The beginning of research of ADAF-models was put by the work, (Narayan & Yi 1994), in which self-similar solving of stationary accretion are constructed. The radial advection stream acts as the basic mechanism of energy

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transferring, and the transference of the angular moment is provided by α-viscosity. The structure of flow appears to be close to spherical and the rotation of gas essentially differs from Keplerian's law. At the presence of strong viscosity ($\alpha \gtrsim 0.3$) numerical modelling yields results which are similar to ADAF-models (Igumenshchev & Abramowicz 2000). However in calculations with small value of parameter α flows are formed with strong turbulence because of development of convective instability, and this instability essentially changes the spatial structure of current (Stone et al. 1999; Igumenshchev & Abramowicz 2000). Such models are called as "convection-dominanted accretion flows" (CDAF) (Balbus & Hawley 2002).

The major properties of the given models are the generation of convective turbulence, transference of the angular moment to the centre, which compensates a stream of the angular moment outside due to viscosity (or, for example, due to magnetic-rotation instability), together with a stream of thermal energy along radius. Numerical modelling shows the low rate of an accretion and a significant stream of energy outside due to a strong radial convection. The fundamental problem of models CDAF is the presence of a stream of the angular moment into center (Balbus & Hawley 2002). The basic results are obtained in frameworks of axisymmetric models (Igumenshchev & Abramowicz 1999; Stone et al. 1999; Balbus & Hawley 2002). The transition to three-dimensional ADAF-models, apparently, does not change decisions of axisymmetric calculations in qualitative sense (Igumenshchev et al. 2000). It is necessary to note, that the important result concerning with convective transference of the angular moment inside was revealed in the models of the rotating stars with a convective nucleus (Bisnovatyi-Kogan et al. 1979).

The presence of magnetic field (MHD CDAF) essentially can change the properties of flow, and, in particular, the convection can result in a stream of the angular moment as inside, and periphery along radial coordinate, and, apparently, it is connected with the influence of magnetic-rotation instability (Igumenshchev 2002; Balbus & Hawley 2002). The model of gas axisymmetric thick disk with the presence of magnetic field shows, that perturbations with the wave length exceeding a vertical scale of a disk, remain convective unstable (Narayan et al. 2002).

It should be noted that the internal radiative-dominant areas of AD can be unstable concerning to the vertical convection and the disk which is thin enough (Bisnovatyi-Kogan et al. 1979). The nonlinear stage of such convection in thin axisymmetric AD is investigated for the standard model radiative-dominant zone for $r - z$-perturbations in work (Agol et al. 2001).

Apparently, the fundamental problem of the AD is the question on nature of turbulent viscosity (Bisnovatyi-Kogan & Lovelace 2002) and, despite of the great progress, active investigation of turbulence in accreting systems is only at the beginning, and the basic results are still more ahead. There is a point of view, that turbulence in the AD is caused by the development of hydrodynamical instabilities at a nonlinear stage and the analysis of unstable flows is an important problem.

In this article we are going to discuss the consequences of development of convective instability in planes of a nonaxisymmetric disk. In this case the reason of nonaxisymmetric convection is connected with the radial non-homogeneity of thermodynamic parameters in quasi-Keplerian disk. We have analyzed the nonlinear dynamics of convective unstable perturbations because of the radial non-homogeneity of entropy. The opportunity of formation of the convective weak turbulence is shown. Convective intermixing in the plane of a disk can result, on the average, to radial accretion without taking into account the action of viscous forces.

2. Basic equations

The equations describing dynamics enough of a thin gas disk should be written down, in the following form:

$$\frac{\partial \sigma}{\partial t} + \frac{\partial (r u \sigma)}{r\,\partial r} + \frac{\partial (v \sigma)}{r\,\partial \varphi} = 0 \,, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \varphi} - \frac{v^2}{r} = -\frac{\partial p}{\sigma\,\partial r} - \frac{\partial \Phi}{\partial r} - D\,\frac{p}{\sigma}\,\frac{d\ln\Omega_z}{dr} \,, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \varphi} + \frac{uv}{r} = -\frac{\partial p}{\sigma\, r\,\partial \varphi} \,, \tag{3}$$

where σ is surface density, p — surface pressure, u and v are radial and azimuthal components of the velocity, respectively, Φ is potential, and the last component in (2) is connected with the averaging of the equations along vertical coordinate (Gorkavyj & Fridman 1994; Khoperskov & Khrapov 1999) and D depends on the features of vertical distribution of gas in a disk, $\Omega_z^2 = \frac{\partial^2 \Phi}{\partial z^2}_{|z=0}$ and in case of Newton's potential for mass M_1 we have $\Omega_z = \Omega_K = \sqrt{GM_1/r^3}$. The equation on pressure should be added to the system of equations (1)–(3)

$$\frac{dp_g}{dt} + \left(1 + 2\,\frac{\gamma-1}{\gamma+1}\right) p_g \nabla \mathbf{v} + 7\,\frac{\gamma-1}{\gamma+1}\left\{\frac{dp_r}{dt} + \frac{9}{7}\,p_r \nabla \mathbf{v}\right\} = 2\,\frac{\gamma-1}{\gamma+1}\,p\,(\mathbf{v}\nabla)\ln(c\,\Omega_z) \,, \tag{4}$$

where p_g and p_r are gas pressure and radiation pressure, γ is an adiabatic index, and the right part appears from the averaging on z-coordinate and the value of parameter c is expressed through D.

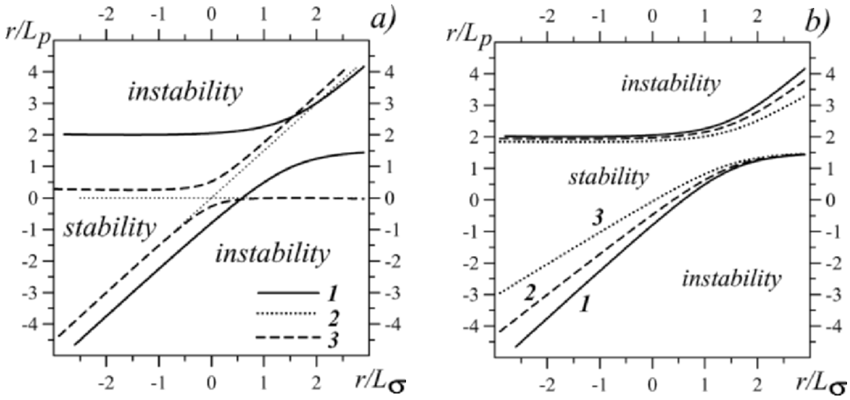

Figure 1. The borders of convective linear instability. *a*) The curve *1* — $r/L_\Omega = r/L_z = -3/2$, $\gamma = 5/3$, $\Lambda = 10$; *2* — unrotative atmosphere (8); *3* — for case $r/L_z = 0$. *b*) *1* — $\gamma = 5/3$, *2* — $\gamma = 4/3$, *3* — $\gamma = 1.01$ for $r/L_\Omega = r/L_z = -3/2$, $\Lambda = 10$.

3. Linear stability analysis

Let's consider a stationary equilibrium non-homogeneous disk without radial motion. The equation (2) gives the balance of radial forces:

$$\frac{v_0^2}{r} - \frac{d\Phi}{dr} - \frac{dp_0}{\sigma_0\, dr} - D\,\frac{p_0}{\sigma_0}\frac{d\,\ln\,\Omega_z}{dr} = 0\,, \qquad (5)$$

where we shall mark equilibrium parameters by an index "0". The two last terms give the small contribution to balance, but the account of pressure gradient $p_0(r)$ is necessary for the development of convection in disk plane. It is convenient to enter specific equilibrium force, for which, taking into account the equation (5), we shall write down

$$g \equiv \frac{d\Phi}{dr} - \frac{v_0^2}{r} + D\frac{p_0}{\sigma_0}\frac{d\ln\Omega_z}{dr} = -\frac{dp_0}{\sigma_0\, dr}\,. \qquad (6)$$

For equilibrium parameters $f = \{\sigma_0, p_0, v_0, \ldots\}$ we shall use the scales of radial non-homogeneity $L_f = (d\,\ln(f)/d\,r)^{-1}$.

Let's consider the dynamics of nonaxisymmetric perturbations, taking into account the non-homogeneous distributions $\sigma_0(r)$, $p_0(r)$, $\Omega = v_0(r)/r = \Omega_z(r)$, and present all the functions as $f(r,\varphi,t) = f_0(r) + \tilde{f}(r,\varphi,t)$. We linearize the equations (1) – (4) concerning the perturbing functions $\tilde{f}$ with the account (6). Within the framework of WKB-approximation it is counted that $\tilde{f} = f_1 \cdot \exp\{-i\,\omega t + i\,kr + i\,m\varphi\}$. Thus, we have the system of four linear algebraic equations concerning amplitudes f_1. The condition of existence of untrivial decisions for this system results in the dispersion equation of 4-th degree concerning frequency ω. If we consider separately the radiative-dominant disk $p_r \gg p_g$ or the other case $p_r \ll p_g$, then the dispersion equation results in:

$$\hat{\omega}^4 - \hat{\omega}^2 \cdot \left[æ^2 + k^2 c_s^2 \cdot \left(1 - 2\,s^2\, \frac{r\, d\Omega^2}{æ^2\, dr} \right) \right] - 2\,s\,\Omega \hat{\omega} k c_T^2\, \Gamma \times$$
$$\frac{d}{dr} \ln \left(\frac{2\Omega\, p_0^{2/\Gamma}}{æ^2 \sigma_0}\, \Omega_z^{\frac{2(1-\Gamma)}{\Gamma}} \right) - \left(\frac{2s\,\Omega k c_T^2}{æ} \right)^2 \frac{d\, \ln(p_0 \Omega_z)}{dr} \cdot \frac{d}{dr} \ln \frac{p_0}{\sigma_0^\Gamma \Omega_z^{\Gamma-1}} = 0, \tag{7}$$

where $\hat{\omega} = \omega - m\Omega$, $\Gamma = 1 + 2(\gamma-1)/(\gamma+1)$ plays a role of a flat parameter of an adiabatic curve, $s = k_\varphi / \sqrt{k^2 + k_\varphi^2}$ defines the degree of perturbations nonaxisymmetry, $k_\varphi = m/r$. In the case of $p_r \gg p_g$ it is necessary to consider $\gamma = 4/3$. In a limit of $\Omega_z = \text{const}$, the equation (7) was received in work (Morozov & Khoperskov 1990).

In a limit of adiabatic model $\frac{p_0}{\sigma_0^\Gamma \Omega_z^{\Gamma-1}} = \text{const}$, the order of the equation (7) reduced is lowered, as the entropy mode degenerates into $\hat{\omega} = 0$. The equation (7) describes two high-frequency acoustic modes for which it is possible approximately to write down $\hat{\omega}^2 \sim æ^2 + k^2 c_s^2$, and two low-frequency modes (entropy and vortical).

Formal transition in (7) to the unrotative medium $\Omega = 0$ and $\Omega_z = \text{const}$ gives $\omega^2 = 4 \frac{s^2}{\Gamma^2} \frac{1}{L_p} (\frac{\Gamma}{L_\sigma} - \frac{1}{L_p})$ for low-frequency waves and for stability it is necessary:

$$\frac{1}{L_p} \left(\frac{\Gamma}{L_\sigma} - \frac{1}{L_p} \right) > 0\,. \tag{8}$$

And it, in accuracy coincides with the condition of convective stability in the non-homogeneous unrotative atmosphere.

Thus, the equation (7) allows to define borders of convective instability with the account of differential rotation ($1/L_\Omega \neq 0$) and the finite thickness of disk in the main approximation ($1/L_z \equiv d\, \ln \Omega_z / dr \neq 0$). Let's write down (7) for low-frequency waves ($|\omega^2| \ll æ^2$):

$$\begin{aligned} \Lambda \cdot \nu^2 + \delta \cdot s \cdot \left[\frac{2r}{\Gamma L_p} - \frac{r}{L_\sigma} - 2\frac{\Gamma-1}{\Gamma}\, \frac{r}{L_z} + \frac{r}{L_\Omega} - 2\, \frac{r}{L_æ} \right] \cdot \nu \\ + s^2\, \frac{\delta^2}{\Gamma^2} \cdot \left[\frac{r}{L_p} - \Gamma\, \frac{r}{L_\sigma} - (\Gamma - 1)\, \frac{r}{L_z} \right] \cdot \left[\frac{r}{L_p} + \frac{r}{L_z} \right] = 0\,, \end{aligned} \tag{9}$$

where $\nu = \hat{\omega}/\Omega$, $\Lambda = \dfrac{æ^2}{\Omega^2} + \dfrac{k^2 c_s^2}{\Omega^2} \cdot \left[1 + 6s^2 \right]$, $\delta = 2kc_s^2/\Omega^2\, r$). Taking into account the estimation for half-thickness of disk $h \sim c_s/\Omega$, parameter Λ can

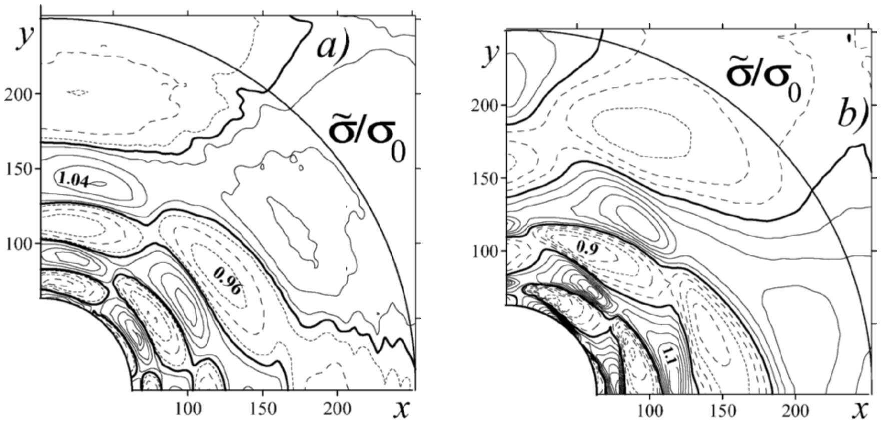

Figure 2. Contours of surface density σ/σ_0 at two different moments of time $t_1 = 15000$ (a), $t_2 = 41000$ (b).

accept values from $\Lambda \simeq 1$ in a long-wave limit, up to $\Lambda \lesssim 100$ for perturbations whose wave length is comparable to thickness of the disk. For many stationary models of AD it is possible to accept the power characteristic of equilibrium parameters of a disk on radial coordinate (Shakura & Sunyaev): $r/L_f = \mathrm{const}$. The condition $\mathrm{Im}(\nu) > 0$ gives the unstable solutions, considered in work (Morozov & Khoperskov 1990) in a case $r/L_z = 0$.

In fig. 1 on a plane of parameters r/L_σ and r/L_p the borders of convective instability determined from a condition $\mathrm{Im}(\nu) = 0$, for base model $r/L_\Omega = r/L_z = -3/2$, $\gamma = 5/3$, $\Lambda = 10$ are represented. Here for comparison there are borders of stability for the unrotative medium for which, the balance is provided only by the external force and the pressure gradient, and for model $r/L_z = 0$. As we see, the rotation and the finite thickness of disk appreciablly change the conditions of convective instability. And, depending on values of L_σ and L_p, the zones of stability on planes $(r/L_\sigma, r/L_p)$, can both be increased and decreased. For equilibrium distributions with $r/L_\sigma < 0$ and $r/L_p < 0$ differential rotations and final thickness of disks are stabilizing factors.

Influence of parameter γ on the condition of convective instability is shown in fig. 1a. At any values of r/L_σ with diminution of γ critical value $|r/L_p|$ becomes less. This effect is corresponded to criterion (8). The parameter Λ characterizes a spatial structure of perturbations. Large values of the parameter $\Lambda \gg 1$ are reached for short-wave waves in radial direction $k \gtrsim \Omega/c_s$ and Λ is higher for perturbations with large azimuthal number m. It is necessary to emphasize, that from the point of view of the equations (9), (7) the most unstable waves are the extremely nonaxisymmetric perturbations $s = 1$ (as $\mathrm{Im}\,\omega \propto s$), for which the made approximations are broken certainly. The

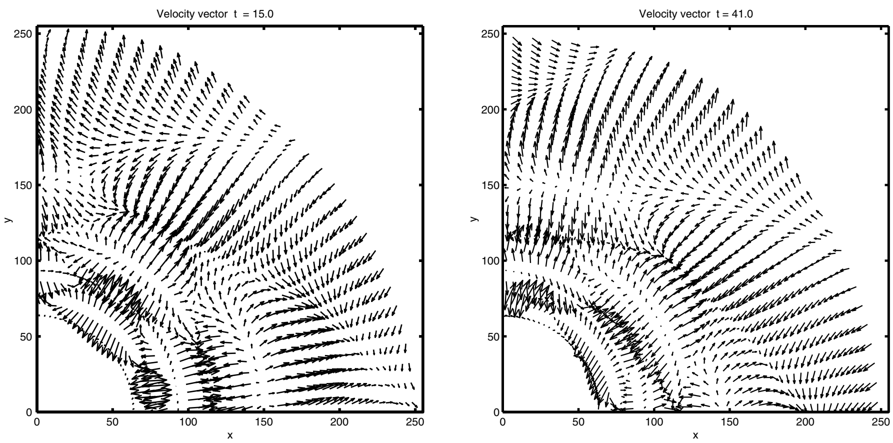

Figure 3. Fields of velocity in a disk plane for the model represented on fig. 2: (a) — $t_1 = 15000$, (b) — $t_2 = 41000$. The length of arrows is proportional to the logarithm of the velocity modulus.

borders of convective instability in disk plane depend on the non-axisymmetry degree of perturbations only through parameter $\Lambda(\Theta)$. The parameter Λ influences on model with $r/L_p > 0$ most of all. Thus models with $r/L_p > 0$ can be convective unstable only concerning small-scale disturbations.

The presence of extreme case (8) affirms the fact, that the physical mechanism causing the growth of perturbations with the time, is similar to classical convective instability at presence of a gradient entropy, which is co-directed to external force. The Archimed's force of buoyancy leads to the convective motion. However, in our case the effects of rotation play the important role. The basic question demanding deep analyze and studying is the influence of strong differential rotation on convective cells at a nonlinear stage.

4. Nonlinear stage of a radial convection

Let's consider the problem on influence of strong differential rotation on convective cells. For solving the equations of hydrodynamics we use the method TVD-E, having limited by $\Omega_z = \mathrm{const}$.

Our model has free parameters: $\delta_p = r/L_p$, $\delta_\sigma = r/L_\sigma$, γ, $\mathcal{M} = r\Omega/c_s$. At the initial moment of time we set the power characteristics of density and pressure $(r/L_p = \mathrm{const}, r/L_\sigma = \mathrm{const})$. Dimensionless coordinates and time: $t = 1$ — a cycle time on radius $r = 1$ should be used. On the external border of settlement area r_{ex} conditions of free course of substance are used. On the internal border r_{in} the conditions of solid wall are used, considering, that the disk reaches the surface of accreting stars in case of a neutron star or

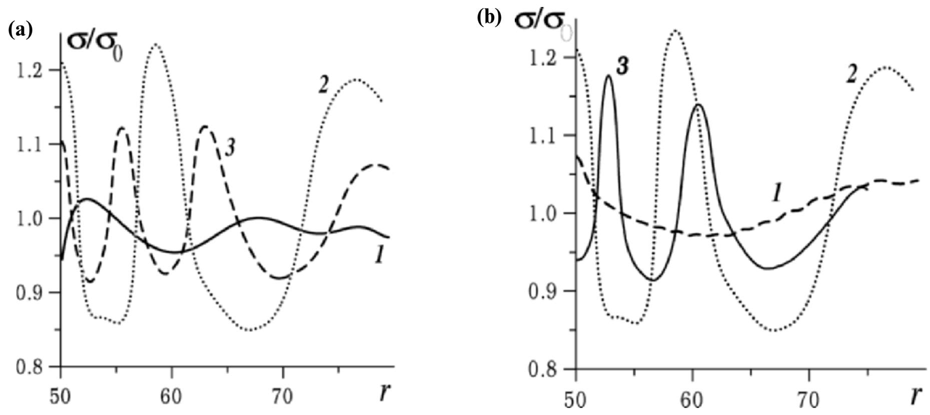

Figure 4. The radial distributions of relative density σ/σ_0. (a) $\mathcal{M} = 10$. The perturbations differ by azimuthal structure: 1 — $m = 4$, 2 — $m = 8$, 3 — $m = 12$. (b) For fixed $m = 8$ models differ by the Mach number $\mathcal{M}$: 1 — $\mathcal{M} = 5$, 2 — $\mathcal{M} = 10$, 3 — $\mathcal{M} = 15$.

the white dwarf. Let's consider only a limit $p_{gas} \gg p_{rad}$, that is carried out at $r_{in} \gg r_G$ ($r_G = 2GM/c^2$ — gravitational radius).

We are going to analyze the dynamics of the waves which are differentiated by azimuthal number m, depending on initial spatial structure of perturbations. For the formation of the certain harmonic m the sector of a disk on a corner φ is considered only. In this case along azimuthal coordinate periodic boundary conditions are used.

Structure of a convection at a nonlinear stage. If the equilibrium condition which is determined by functions $p_0(r)$ and $\sigma_0(r)$ provides stability of a disk according to (7) ($\mathrm{Im}(\omega) = 0$) the fact is that the increasing of perturbations in due course does not occur in numerical models. Control calculation in case of $r/L_p = -2$, $r/L_\sigma = -1$ shows, that on an extent $t \leq 10^5$ at the presence of initial perturbations with initial amplitude $\lesssim 2\%$ their further increase does not occur.

Let's consider the model with $r/L_p = -3/2$ and $r/L_\sigma = -1/2$, $\gamma = 5/3$ which gets into the unstable area according to (9). Regardless of amplitude of initial perturbation, we obtain typical spiral-cellular wave structure.

In fig. 2 there are contours of ratio of density $\sigma(r, \varphi)$ to equilibrium value $\sigma_0(r)$ for two moments of time $t_1 = 15\,000$ and $t_2 = 41\,000$. At the initial stage typical convective cells (see fig. 2 a) with small relative amplitude of surface density $|\sigma - \sigma_0|/\sigma_0 \lesssim 0,05$ are formed. With time the increasing of amplitude and the complication of spatial structure because of the differential rotation of disk take place (see fig. 2 b).

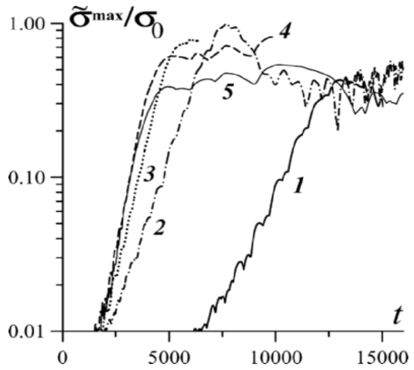

Figure 5. Dependences of the maximal value of the perturbed density $\tilde{\sigma}^{\max}/\sigma_0$ for a convective cell from time for various models: (a) $\mathcal{M} = 10$, 1 — $m = 8$, 2 — $m = 12$, 3 — $m = 20$. (b) For fixed m models differ by number of the Mach $\mathcal{M}$: 1 — $\mathcal{M} = 10$, 2 — $\mathcal{M} = 20$, 3 — $\mathcal{M} = 40$.

The law of rotation differs from Keplerian Ω_K just a little. With the increasing of $\mathcal{M}$ these deviations are decreasing. Since the full radial component of velocity u and the perturbation of azimuthal velocity $\tilde{v} = v - v_0$ are rather small in comparison with the equilibrium velocity of rotation $v_0 \simeq r\Omega_K$ it is more convenient to consider only the perturb components but not a full field of velocities. In fig. 3 the vector field of velocity perturbations is represented in a plane of a disk. The field of velocities demonstrates vortical character of flow.

The spatial structure of perturbation can be characterized by radial wave number k and by azimuthal number m, which are independent within the framework of the linear analysis. The radial structure of unstable perturbations at the nonlinear stage is defined by parameters of model ($\mathcal{M}$, m, δ_p, δ_σ, γ), and we can change azimuthal number m, varying the initial perturbations along angle φ. In fig. 4 it is visible, that with the increasing of azimuthal number the perturbations become more small-scale in the radial direction as well. There are similar effect in the case of increasing number of the Mach ($\mathcal{M}$).

It follows from the linear analysis, that the increment of instability is proportional $\mathrm{Im}\,\omega \propto m$, and as a whole this fact is affirmed at an initial stage of evolution of perturbations. In fig. 5 time dependences of amplitude of relative density for the chosen convective cell for various m and $\mathcal{M}$ are shown. During the typical times $t^{(sat)} \simeq 350\mathcal{M}$ the increasing of perturbations up to much nonlinear stage, close to saturation, takes place. The disturbance amplitude with small azimuthal number arises slowly.

At the non-linear stage of instability the spiral shock waves (SW) are formed in a disk, and it is caused by supersonic flowing of gas onto the convective cells. In fig. 6 the structure of shock waves is shown. The distributions of $(\mathrm{div}\,\vec{\mathbf{v}})^2$

most evidently demonstrates positions of fronts of shock waves in fig. 6. Radial structure of parameters of gas at the fixed values of an azimuthal angle φ are represented in fig. 7. The shock wave is formed on the back edge of the spiral density perturbation, and the wave of underpressure is observed on the front edge.

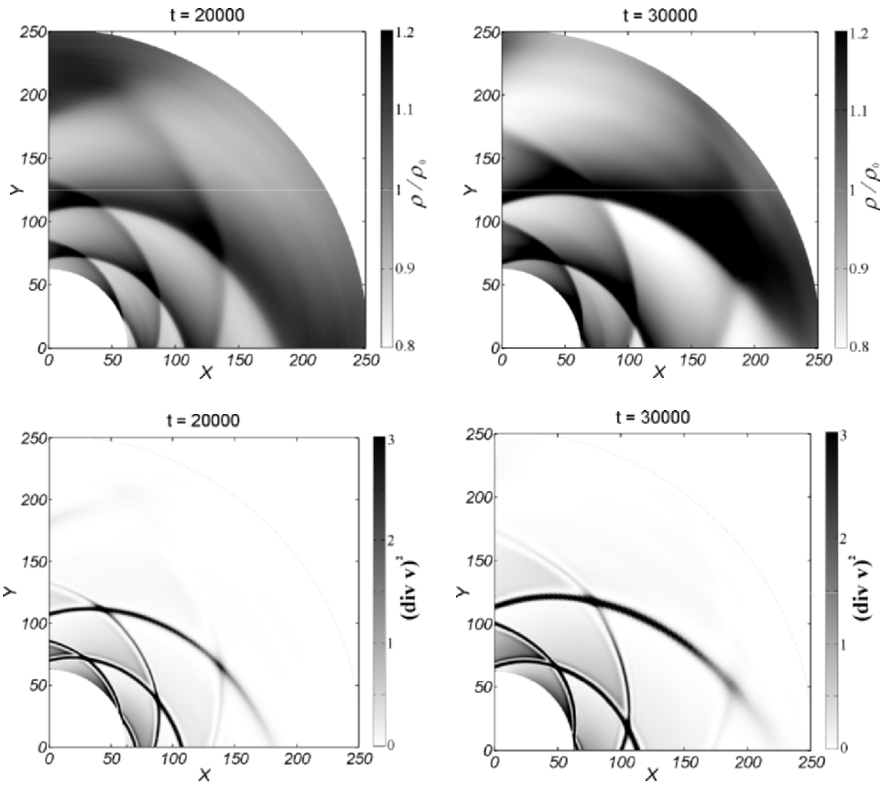

Figure 6. Contours of the relative density and $(\mathrm{div}\,\vec{\mathbf{v}})^2$ for the two different moments of the time.

Is viscosity necessary for an accretion? The result of convection in disk plane is gas falling onto the gravitational center, on average.

In fig. 8 time dependences of the current of mass $\dot{M} = r\int_0^{2\pi}\sigma u\, d\varphi$ are represented on three various radiuses. Non-stationary character of accretion is connected with the absence of the stationary solution because of used boundary conditions in radius r_{in}. The condition of solid wall on inner boundary results in the accumulation of mass in a disk that has an effect for other parameters as well (fig. 9). The current of the angular moment is directed outside, that

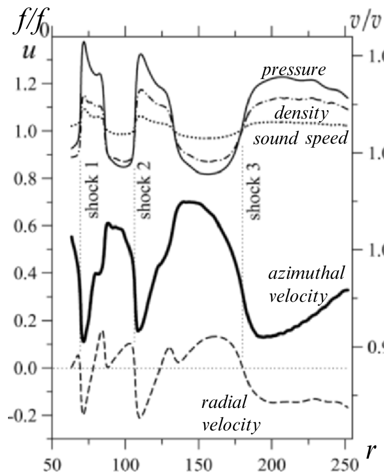

Figure 7. The radial dependences σ/σ_0, p/p_0, c_s/c_{s0}, u (left axis), v/v_0 (right axis) for the azimuthal angle $\varphi = 0$ at time $t = 2 \cdot 10^4$. The positions of shock waves are indicated.

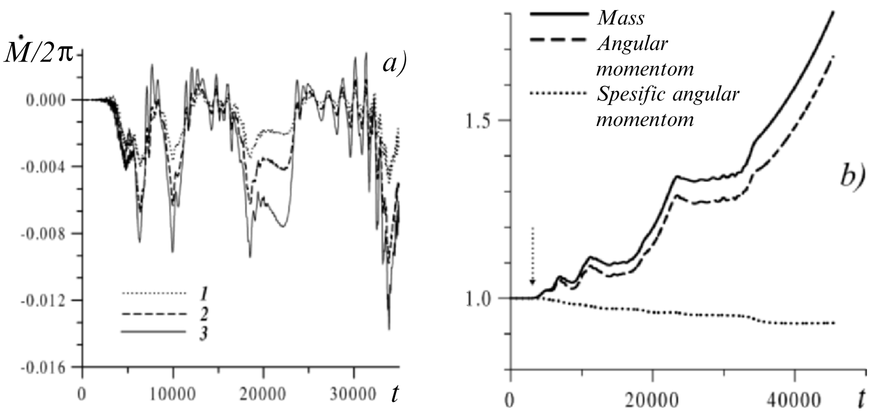

Figure 8. *a*) The dependences $\dot{M}$ from time through three various circles of the fixed radius. *b*) The time dependences of integral values (mass M, angular moment $\mathcal{L}$ and specific angular moment $l = \mathcal{L}/M$) in calculated region of numerical model.

appears to be the important distinctive feature from models CDAF (Balbus & Hawley 2002). Such received direction of the current affirms, that rotation is a source of free energy (8).

In fig. 9 the radial distributions of relative sound speed in a disk at the different time moments are shown. Monotonous increase of c_s in due course points out to the heating of a disk as a result of gravitational energy release.

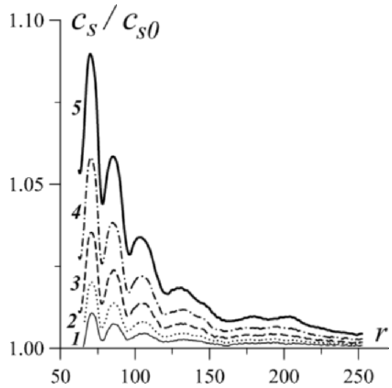

Figure 9. The dependence $c_s(r)/c_{s0}(r)$ at the different time moments (the value c_s is received as the result of averaging on a angle φ, and c_{s0} is initial equilibrium value): 1 — $t_1 = 30\,000$, 2 — $t_2 = 35\,000$, 3 — $t_3 = 40\,000$, 4 — $t_4 = 45\,000$, 5 — $t_5 = 50\,000$.

The increasing of temperature in the internal part of a disk is higher, than in periphery.

In summary, it should be noted, that the received results, made in frameworks of quasi-Keplerian disk, are kept, on the whole, in case of rotation laws with $r/L_\Omega > -3/2$. In the appendix to the gas subsystems of disk galaxies, the values of parameter r/L_Ω lay from 0 (close to solid-body rotation in the central zone) up to -1, and the curve of plateau-type is characteristic for the majority of galaxies in the most part of a disk.

5. Discussion of results

We have studied nonlinear dynamics of convective unstable perturbations because of radial non-homogeneity of entropy, neglecting effects of cooling and viscosity. Unlike numerous works on studying of convection in a plane $r - z$, we have shown principled opportunity of development of convective instability in thin quasi-Keplerian disk, where vertical motion (if they are present) are not a reason of the convection.

The opportunity of formation of the weak convective turbulence in characteristic times $\sim(10^3 \div 10^4)\,\tau$ in the central zone of disk (τ — Keplerian period on radius $3r_g = 6\,GM/c^2$) is shown. The rate of convection generation is increased for the peripheral region of disk. The convective intermixing in disk plane can result in average radial current of mass without taking into account action of viscous forces.

The convective instability considered here in a plane of a disk is not connected with the fact of gas rotation, as in case of fluid motion between two rotating cylinders (Taylor flow), that results in the formation of Taylor vortex. The opportunity of convection development in the disk plane is kept and in the case of solid-body rotation $L_\Omega^{-1} = \frac{d\Omega}{\Omega\, dr} = 0$, and it can have interest for models of gas disks of galaxies. The convection conditions can be fulfilled for the curve of rotation of galactic gas disks such as a plateau $V = r\Omega = \mathrm{const}$. In contrast to the convection in the appendix to stars (Brun & Toomre 2002), the degree of differential rotation in the AD are much higher. The physical reason of such instability is the decrease of specific entropy with radius. The role of external force which is required for convection development is played by the value $g \simeq \frac{\partial\Phi}{\partial r} - r\Omega^2$, and in the case of $\frac{ds_0}{dr} < 0$ it is necessary $g > 0$. It should be noted, that instability is possible even at $g < 0$ in the case of $\frac{ds_0}{dr} > 0$.

Acknowledgments

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