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Beats of Magnetohydrodynamical and Rossby Waves and their Possible Effect on the Formation of Solar Cyclicity

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Abstract—We analyze beats of slow magneto-acoustic and Rossby waves propagating in different directions as a possible mechanism of the formation of the magnetic cyclicity of the Sun. The dispersion law derived in terms of linear magnetohydrodynamics unambiguously indicates the presence of beats in the system. We demonstrate that the periods of such beats vary from 9 to 13 years.

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1. INTRODUCTION

Solar activity and, in particular, the phenomenon of sunspots, has been attracting the researchers' attention for more than half a century. It has long become evident that sunspots form when matter emerging from lower levels onto the boundary between the photosphere and chromosphere carries intense magnetic flux with it. This matter is compressed by the pressure of ambient plasma, and magnetic field and hence magnetic pressure inside it are greater than in the surrounding regions, implying that the contribution of thermodynamical pressure to the total pressure in the sunspot region is substantially less than in its neighborhood. Convection is suppressed by the strong magnetic field and therefore the temperature in the emerging matter is substantially lower than the temperature in the ambient plasma; as a result, the brightness in the spot region is lower.

At the same time, active regions are known to be usually observed at intermediate and low latitudes. Moreover, the regularity of their appearance and their quasi-periodic (active longitudes) distribution in the latitude direction suggest that large-scale collective wave processes should be responsible for their formation. We believe the search for, identification and study of such processes to be tasks of prime importance.

Rossby waves and vortices are large-scale perturbations in rotating gaseous and liquid systems. Waves of this class owe their existence to the nonuniform dependence of rotation velocity along the meridian or radial direction—in the case of thin disks—and to the shear elasticity of the medium due to the specific distribution of Coriolis forces. A characteristic feature of such waves is that the time scales of wave motions are longer than the period of rotation of the system. Given that the overwhelming majority of astrophysical objects have substantial angular momenta and developed gaseous subsystems, the class of perturbations considered should play important part in their dynamics and evolution.

Even so, the class of perturbations discussed remains outside the scope of most of the studies.

Note that Lou [1] tried to invoke Rossby waves to explain the formation of sunspots, however, his attempts did not go beyond the simple statement of the fact.

Before going further, let us mention some bona fide facts concerning planetary atmospheres and oceans [2–5], which are of importance in the context of this paper:

- The equation describing the dispersion law in planetary atmospheres and oceans derived in terms of shallow-water theory has the following form in the Cartesian coordinate system corotating with the planet:

$$\omega(\omega^2 - 4\Omega_z^2 - k_\perp^2 c_s^2) - \frac{2k_x\Omega_y}{R}c_s^2 = 0, \qquad (1)$$

where ω is the frequency; Ω_z, the projection of the system's rotation velocity onto the local vertical; Ω_y, the projection of the system's rotation velocity on the meridian; c_s, the adiabatic sound speed; R, the radius of the planet; $k_\perp^2 = k_x^2 + k_y^2$; k_x, the wavenumber along the latitude direction, and k_y, the wavenumber along the meridian. The high–frequency solution of this equation ($\omega \geq 2\Omega_z$) gives the dispersion law for gravity-gyroscopic waves and the low-frequency solution, the dispersion law for Rossby waves:

$$\omega_R = -\frac{2k_x\Omega_y}{R(k_\perp^2 + 4\Omega_z^2/c_s^2)}; \qquad (2)$$

- The rotation of the system affects substantially the dynamics and properties of wave structures if the so-called Rossby mode (effect) is satisfied, namely: $l_\perp \gg v/(2\Omega_z)$ or $Ro = v/(2\Omega_z l_\perp) \ll 1$, where $l_\perp$ is the scale length of the structure in the plane perpendicular to the local vertical; v is the characteristic velocity of wave motions, and Ro is the Kibel–Rossby number (the small Kibel–Rossby number is a necessary condition for the rotation of the system to have an important effect on the properties of the structures studied). Motions in the wave are subsonic in all cases and therefore the sufficient condition for the Rossby mode has the form $l_\perp \geq r_R = c_s/(2\Omega_z)$, where r_R is the Rossby–Obukhov radius (the Rossby–Obukhov radius is the natural spatial scale length in the large-scale dynamnics of the atmosphere);

- Long-wavelength perturbations at low latitudes due to the Coriolis force and its nonuniform distribution along the meridian are actually Rossby waves (planetary waves), which in the course of the nonlinear stage maintain or produce zonal (i.e., latitudinal) flows (vortices) at low latitudes and regularly alternating cyclonic and anticyclonic Rossby vortices at midlatitudes with the vorticity of velocity parallel or antiparallel to the vector of local angular velocity of rotation, respectively. This case is qualitatively similar to the well-known problem of electric drift of particles in crossed electric and magnetic fields (see, e.g., [6]))—equatorial planetary waves are responsible for the motion along the trachoid without loops and Rossby waves, for the motion along the trachoid with loops; there is also a strict analogy—up to redesignation of the parameters in the dispersion law—with drift waves in magnetized plasma, where electron temperature is much higher than the ion temperature [3]). Other characteristic examples of cyclonic and anticyclonic vortices include the so-called "barges" in the atmosphere of Jupiter; Jupiter's Great Red Spot, which is actually a Rossby autosoliton, and a similar autosoliton on Neptune;

- In cyclones the Coriolis force is directed away from the center of the vortex and in anticyclones, toward the center, and therefore gas density decreases in cyclones and increases in anticyclones;

- Anticyclones live much longer than cyclones because of the dispersion peculiarities (note, by the way, that because of the density increase, other conditions being equal, the total angular momentum of an anticyclone is greater than the that of a cyclone, and therefore the former disrupts less easily);

- Rossby vortices slowly drift westward along the latitude direction at a velocity that does not exceed $V_{dr} \simeq V_R$, where $V_R = \omega_R/k_\perp$ is the phase velocity of Rossby waves determined in terms of linear analysis—see (2);

- The most interesting Kibel–Rossby number interval from the application viewpoint can be limited from below based on the considerations of maximal nonlinearity of the Rossby wave development mode so that the particles of the medium would be captured and drift alongside waves or vortices: $Ro > r_R/R$, where R is the radius of the planet;

- The larger is the size of the system the better are conditions or the Rossby mode satisfied; its manifestations on giant planets are much more conspicuous than under terrestrial conditions, and this fact leads us to suspect that this mode may exist under the conditions found on stars.

In Section 1 we discuss the observed facts, which associate Rossby waves with solar active regions; in Section 2 we describe a stationary model; in Section 3 we give the main equations and their linearized versions, the sought-for dispersion equation in the general and special cases. In Section 4 we discuss the results obtained and in Section 5, we summarize the main conclusions.

2. RELATIONSHIP BETWEEN ACTIVE REGIONS AND ROSSBY WAVES

We now mention some observational facts, which are indicative, in our opinion, of the possible development of Rossby waves under solar conditions:

(1) Localization of active regions mostly at mid- and low latitudes;

(2) Commonly encountered periodicity or quasiperiodicity of active regions in longitude (also evidenced by SOHO data, which are available from `http://sohowww.nascom.nasa.gov/`);

(3) Sunspot drift velocity is comparable to the characteristic drift velocity of Rossby vortices: $v_s \simeq v_R$;

We can derive from equation (2) for solar conditions, but ignoring magnetic-field effects:

$$v_R = \frac{\omega_R}{k_\perp} = -\frac{2R_\odot \Omega \cos\theta}{(k_\perp^2 R_\odot^2 + 4R_\odot^2 \Omega^2 \sin^2\theta / c_s^2)} \cos\varphi, \quad (3)$$

where φ is the angle between latitude (as a heliographic line) and vector $k_\perp$.

Based on the typical solar parameters—the radius $R_\odot \simeq 6.9599(7) \times 10^8$ m, adiabatic sound speed $c_s \leq$ 6 km/s (which corresponds to temperatures below 6000 K), and angular rotation velocity of $\Omega \simeq 2.865 \times 10^{-6}$ rad/s (corresponding to the rotation period of 25.38 days at a latitude of $\theta \simeq 17°$), we find that the second term in the denominator is very small, because $M^2 = R_\odot^2 \Omega^2 / c_s^2 \simeq 0.11$. At the same time, the minimum value of the first term (i.e., if $k_\perp$ is perpendicular to meridian) is not small. In this case we find for perturbations with m wavelengths along the latitude direction:

$$k_\perp R_\odot = \frac{2\pi R_\odot}{2\pi R_\odot} \frac{m}{\cos\theta} > m, \quad (4)$$

and, correspondingly, obtain for $m \geq 6$ $k_\perp^2 R_\odot^2 \geq 36$.

We thus derive the following rather accurate approximation:

$$v_R \simeq -\frac{2R_\odot \Omega \cos\theta}{m^2} \cos\varphi. \quad (5)$$

At the same time, according to observational data, sunspots move by 0.3–0.6 heliocentric degrees every day, and the drift occurs mostly westward. Hence the sunspot velocity (in m/s) can be estimated as:

$$v_s \simeq -\frac{(0.3\ldots0.6)}{360} \times \frac{2\pi R_\odot \cos\theta}{24 \times 3600}. \quad (6)$$

We now divide formula (6) by formula (5) to find, by adopting $m = 6$, that $v_s/v_R \simeq (0.38\ldots0.76)/\cos\varphi$. Hence the velocities discussed remain rather close to each other over a wide interval of angles $\varphi \simeq 0° - 70°$.

We now give a number of other estimates. First, we determine the Rossby–Obukhov radius for not too low latitudes: $r_R = c_s/(2\Omega_z) \simeq (10^6 - 10^7)$ km.

For cyclone–anticyclone structures that repeat exactly m times along the latitude the transversal scale of cyclonic vortices is equal to $l_\perp = 2\pi R_\odot \cos\theta/(2m)$, i.e., the sufficient condition for the existence of the Rossby mode at midlaltitudes $l_\perp \geq r_R$ is well satisfied for all $m \leq 10$.

Furthermore, nonlinearity is important if for the Kibel–Rossby number equal to

$$R_o = \frac{v}{2\Omega_z l_\perp} < \frac{c_s}{2\Omega_z l_\perp}, \quad (7)$$

the following condition is satisfied:

$$R_o > \frac{r_R}{R_\odot} = \frac{c_s}{2\Omega_z R_\odot}. \quad (8)$$

We now compare formulas (7) and (8) in view of the formula for $l_\perp$ to find that condition $m/(\pi\cos\theta) > 1$ must be satisfied, which is true for all $m \geq 3$. Hence the formation of Rossby vortices is quite possible under solar conditions.

3. EQUILIBRIUM MODEL

As our equilibrium model we adopt the following modification of the model proposed by Mustsevoi and Solov'ev [7].

We perform our analysis in the local Cartesian coordinate system rotating at an angular velocity of $\mathbf{\Omega}$ (we assume that the Sun rotates as a solid body for the sake of simplicity) with effective gravity $\mathbf{g}$ such that the following condition:

$$\mathbf{g} = \tilde{\mathbf{g}} + \nabla \frac{[\mathbf{\Omega R_\odot}]^2}{2}, \quad (9)$$

is satisfied, where $\tilde{\mathbf{g}}$ is local gravity (without the allowance for rotation) and $\mathbf{R}_\odot$ is the radius-vector pointing from the center of the Sun to the origin of the coordinate system.

Let unit vector $\mathbf{e_z}$ be parallel to the resulting gravity vector $\mathbf{g}$ (with the allowance for rotation). Let the unit vector $\mathbf{e_y}$ be directed northward along the meridian, and unit vector $\mathbf{e_x}$, eastward along the latitude line.

We denote the thermodynamical parameters of photospheric gas (plasma), i.e., density, temperature, and pressure, as ρ, T, and P, respectively (for the quantitative computations below we adopt $T \simeq$ 6000 K, $\rho \simeq 10^{-7}$ g/cm^3, and determine pressure from the equation of state of ideal gas: $P = \rho c_s^2/\gamma$, where $c_s^2 = \gamma\lambda T/\mu$ is the squared adiabatic sound speed; γ, the adiabatic index; λ, the universal gas constant, and μ, the molar mass).

We adopt the atmospheric level characterized (at the bottom) by the above parameters as the tangential discontinuity surface with a vertical coordinate equal to $z = 0$ and with the classic condition of equal total pressure on both sides at the interface between the two layers:

$$P_{in}(0) + \frac{B_{in}^2(0)}{8\pi} = P_{ex}(0) + \frac{B_{ex}^2(0)}{8\pi}. \quad (10)$$

We set the unperturbed velocity of gas flow in the adopted local reference frame equal everywhere to zero.

We finally point out that at the boundary between the photosphere and chromosphere magnetic field cannot have nonzero vertical component, because this would mean continuous variation of magnetic field and gas pressure at the transition across the layer [8]. This, in turn, means that, in view of the axial symmetry and equation $\mathrm{div}\mathbf{B} = 0$, the horizontal component of magnetic field should obey equation $B_\theta = B_{\theta 0}/\cos\theta$, where $B_{\theta 0} = B_{\theta 0}(z)$ is the value of the θ component of magnetic field at the equator and θ is, as before, the latitude. We assume that the azimuthal component $B_{\varphi 0} = B_{\varphi 0}(z)$ is constant in the horizontal plane.

Let us now discuss the vertical balance of forces in the adopted model. Its stationarity is ensured by the following condition:

$$g = -\frac{1}{\rho}\frac{dP}{dz} - \frac{1}{8\pi\rho}\frac{dB^2}{dz}$$
$$= -\frac{1}{\gamma}\frac{dc_s^2}{dz} - \frac{c_s^2}{\gamma}\frac{d\ln\rho}{dz} - \frac{1}{8\pi\rho}\frac{dB^2}{dz}. \quad (11)$$

We assume that the atmosphere is isothermal at $z \neq 0$ (i.e., $c_s = \mathrm{const}$), in good agreement with the observed temperature distribution in the transition layer between the photosphere and chromosphere considered here, and further assume that $B^2(z)/\rho(z) = \mathrm{const}$, to find from equation (11) that:

$$g = -\left[\frac{c_s^2}{\gamma} + \frac{B^2(0)}{8\pi\rho(0)}\right]\frac{d\ln\rho}{dz}. \quad (12)$$

It follows from this that at $g = \mathrm{const}$:

$$\rho = \rho(0)\exp\left(-\frac{\gamma g z}{c_s^2}\right), \quad (13)$$

where

$$c_s^2 + \frac{\gamma B^2(0)}{8\pi\rho(0)} = \mathrm{const}. \quad (14)$$

Note that the particular form of the z distributions of the parameters of the medium in the vicinity of the discontinuity has no crucial effect on the dispersion law provided that these distributions are are smooth at $z \neq 0$ and have a large-scale structure. At the same time, the adopted model is, on the one hand, sufficiently realistic, and, on the other hand, it allows an analytical dispersion law for perturbations to be explicitly derived.

Given that the class of perturbations that is of interest for us is sufficiently large-scale, we determine, to a first approximation, the latitude variations of local angular rotation velocity $\Omega(\theta)$ and squared magnetic-field intensity $B^2(\theta)$. In the absence of more convincing and reliable data we adopt that magnetic-field intensity can be described by the following formula:

$$B = \sqrt{B_\varphi^2 + B_\theta^2} = \sqrt{B_{\varphi 0}^2 + B_{\theta 0}^2/\cos^2\theta}. \quad (15)$$

We now expand it into a Taylor series up to linear terms in small parameters $\theta - \theta_0$ to determine:

$$\Omega_z = \Omega\sin\theta \simeq \Omega\sin\theta_0 + R_\odot(\theta - \theta_0)\Omega\frac{\cos\theta_0}{R_\odot}$$
$$= \Omega_{z_0} + y\frac{\Omega_{y_0}}{R_\odot} = \Omega_{z_0} + f_\Omega y, \quad (16)$$

$$B^2 = B_{\varphi 0}^2 + \frac{B_{\theta 0}^2}{\cos^2\theta_0} + R_\odot(\theta - \theta_0)\frac{2B_{\theta 0}^2\sin\theta_0}{R_\odot\cos^3\theta_0}$$
$$= B^2 + f_B y. \quad (17)$$

4. BASIC EQUATIONS

We proceed from the set of equations of ideal magnetic hydrodynamics:

$$\frac{\partial\mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} + 2[\mathbf{\Omega}\mathbf{V}] - \mathbf{\Omega}^2\mathbf{r}$$
$$= -\frac{1}{\rho}\nabla P + \mathbf{g} - \frac{1}{8\pi\rho}\nabla\mathbf{B}^2 + \frac{1}{4\pi\rho}(\mathbf{B}\nabla)\mathbf{B}, \quad (18)$$

$$\frac{\partial\mathbf{B}}{\partial t} = (\mathbf{B}\nabla)\mathbf{V} - (\mathbf{V}\nabla)\mathbf{B} - \mathbf{B}\,\mathrm{div}\,\mathbf{V},$$
$$\mathrm{div}\,\mathbf{B} = 0, \quad (19)$$

$$\frac{\partial\rho}{\partial t} + (\mathbf{V}\nabla)\rho + \rho\,\mathrm{div}\,\mathbf{V} = 0,$$
$$\frac{\partial P}{\partial t} + (\mathbf{V}\nabla)P = c_s^2\left[\frac{\partial\rho}{\partial t} + (\mathbf{V}\nabla)\rho\right], \quad (20)$$

supplemented by the equation of state of ideal gas $\gamma P = c_s^2\rho$. Here $\mathbf{r}$ is the radius-vector directed from the rotation axis to the point considered ($r = R_\odot\cos\Theta$).

We now use standard linearization procedure to write all quantities in the equation set (18)–(20) in the form $f + \tilde{f}$, where $|\tilde{f}| \ll f$ ($|\tilde{\mathbf{V}}| \ll c_s$ for perturbed velocity). In this case, given that the equations of stationary balance of forces are satisfied and the unperturbed velocity is equal to zero, the initial linearized equation set acquires the following form:

$$\frac{\partial\tilde{\mathbf{V}}}{\partial t} + 2[\mathbf{\Omega}\tilde{\mathbf{V}}] = -\frac{1}{\rho}\nabla\tilde{P} + \frac{\tilde{\rho}}{\rho}\mathbf{g}$$
$$-\frac{1}{4\pi\rho}\nabla(\mathbf{B}\tilde{\mathbf{b}}) + \frac{1}{4\pi\rho}(\mathbf{B}\nabla_\perp)\tilde{\mathbf{b}} + \frac{1}{4\pi\rho}(\tilde{\mathbf{b}}\nabla)\mathbf{B}, \quad (21)$$

$$\frac{\partial \tilde{\mathbf{b}}}{\partial t} = (\mathbf{B}\nabla_\perp)\tilde{\mathbf{V}} - (\tilde{\mathbf{V}}\nabla)\mathbf{B} - \mathbf{B}\mathrm{div}\tilde{\mathbf{V}},$$
$$\mathrm{div}\tilde{\mathbf{b}} = 0, \quad (22)$$

$$\frac{\partial \tilde{\rho}}{\partial t} + (\tilde{\mathbf{V}}\nabla)\rho + \rho\,\mathrm{div}\tilde{\mathbf{V}} = 0, \quad (23)$$

$$\frac{\partial \tilde{P}}{\partial t} + (\tilde{\mathbf{V}}\nabla)P = c_s^2\left[\frac{\partial \tilde{\rho}}{\partial t} + (\tilde{\mathbf{V}}\nabla)\rho\right]. \quad (24)$$

In view of the above, we assume that unperturbed magnetic field is horizontal: $\mathbf{B} = \{B_x,\ B_y\}$. In this case, if written coordinate-wise, equation set (21)–(24) acquires the following form:

$$\frac{\partial \tilde{V}_x}{\partial t} - 2\Omega_z\tilde{V}_y = -\frac{1}{\rho}\frac{\partial}{\partial x}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big)$$
$$+\frac{(\mathbf{B}\nabla_\perp)}{4\pi\rho}\tilde{b}_x + \frac{1}{4\pi\rho}\tilde{b}_y\frac{\partial B_x}{\partial y} + \frac{1}{4\pi\rho}\tilde{b}_z\frac{\partial B_x}{\partial z}, \quad (25)$$

$$\frac{\partial \tilde{V}_y}{\partial t} + 2\Omega_z\tilde{V}_x = -\frac{1}{\rho}\frac{\partial}{\partial y}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big)$$
$$+\frac{(\mathbf{B}\nabla_\perp)}{4\pi\rho}\tilde{b}_y + \frac{1}{4\pi\rho}\tilde{b}_y\frac{\partial B_y}{\partial y} + \frac{1}{4\pi\rho}\tilde{b}_z\frac{\partial B_y}{\partial z}, \quad (26)$$

$$\frac{\partial \tilde{V}_z}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial z}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big) - \frac{\tilde{\rho}}{\rho}g$$
$$+\frac{(\mathbf{B}\nabla_\perp)}{4\pi\rho}\tilde{b}_z, \quad (27)$$

$$\frac{\partial \tilde{b}_x}{\partial t} = (\mathbf{B}\nabla_\perp)\tilde{V}_x - \tilde{V}_y\frac{\partial B_x}{\partial y}$$
$$-B_x\Big(\frac{\partial \tilde{V}_x}{\partial x} + \frac{\partial \tilde{V}_y}{\partial y} + \frac{\partial \tilde{V}_z}{\partial z}\Big), \quad (28)$$

$$\frac{\partial \tilde{b}_y}{\partial t} = (\mathbf{B}\nabla_\perp)\tilde{V}_y - \tilde{V}_y\frac{\partial B_y}{\partial y}$$
$$-B_y\Big(\frac{\partial \tilde{V}_x}{\partial x} + \frac{\partial \tilde{V}_y}{\partial y} + \frac{\partial \tilde{V}_z}{\partial z}\Big), \quad (29)$$

$$\frac{\partial \tilde{b}_z}{\partial t} = (\mathbf{B}\nabla_\perp)\tilde{V}_z. \quad (30)$$

It follows from equation (30) that the unperturbed z component of magnetic field in the case considered depends only on the perturbation of the z component of velocity. Therefore equation set (25)–(30) allows split solutions, and there is a possible class of solutions with $\tilde{b}_z \equiv 0$, $\tilde{V}_z \equiv 0$. Note that this class of perturbations is of greatest interest for us, because such perturbations would be least affected by the absorption in the narrow nonstationary layer between the photosphere and lower chromosphere. In this case equations (25)–(27) reduce to the following form:

$$\frac{\partial \tilde{V}_x}{\partial t} - 2\Omega_z\tilde{V}_y = -\frac{1}{\rho}\frac{\partial}{\partial x}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big)$$
$$+\frac{(\mathbf{B}\nabla_\perp)}{4\pi\rho}\tilde{b}_x + \frac{1}{4\pi\rho}\tilde{b}_y\frac{\partial B_x}{\partial y}, \quad (31)$$

$$\frac{\partial \tilde{V}_y}{\partial t} + 2\Omega_z\tilde{V}_x = -\frac{1}{\rho}\frac{\partial}{\partial y}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big)$$
$$+\frac{(\mathbf{B}\nabla_\perp)}{4\pi\rho}\tilde{b}_y + \frac{1}{4\pi\rho}\tilde{b}_y\frac{\partial B_y}{\partial y}, \quad (32)$$

$$-\frac{1}{\rho}\frac{\partial}{\partial z}\Big(\tilde{P} + \frac{(\mathbf{B}\tilde{\mathbf{b}})}{4\pi}\Big) - \frac{\tilde{\rho}}{\rho}g = 0. \quad (33)$$

The coefficients of the resulting equation set are homogeneous in x and t (they depend neither on x or t) and therefore we look for solutions in the form of plane waves (by expanding them into normal modes): $\tilde{f} = f(y,\ z)\exp\{ik_x x - i\omega t\}$. We then derive from equations (31)–(32):

$$-i\omega V_x - 2\Omega_z V_y = -\frac{ik_x}{\rho}P - \frac{ik_x}{4\pi\rho}B_y b_y$$
$$+\frac{B_y}{4\pi\rho}\frac{\partial b_x}{\partial y} + +\frac{1}{4\pi\rho}b_y\frac{\partial B_x}{\partial y}, \quad (34)$$

$$-i\omega V_y + 2\Omega_z V_x = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{ik_x}{4\pi\rho}B_x b_y$$
$$-\frac{B_x}{4\pi\rho}\frac{\partial b_x}{\partial y} - \frac{1}{4\pi\rho}b_x\frac{\partial B_x}{\partial y}. \quad (35)$$

To take into account the weak dependence of equilibrium parameters on y, we multiply equation (34) by ρ and differentiate it with respect to y. Given that we write this dependence as a series up to terms linear in y, we find:

$$-(i\omega V_x + 2\Omega_z V_y)\frac{\partial \ln\rho}{\partial y} - i\omega\frac{\partial V_x}{\partial y} - 2\Omega_z\frac{\partial V_y}{\partial y} - 2f_\Omega V_y$$
$$= -\frac{ik_x}{\rho}\frac{\partial P}{\partial y} - \frac{ik_x}{4\pi\rho}B_y\frac{\partial b_y}{\partial y} - \frac{ik_x}{4\pi\rho}b_y\frac{\partial B_y}{\partial y}$$
$$+\frac{1}{4\pi\rho}\frac{\partial B_y}{\partial y}\frac{\partial b_x}{\partial y} + \frac{B_y}{4\pi\rho}\frac{\partial^2 b_x}{\partial y^2} + \frac{1}{4\pi\rho}\frac{\partial b_y}{\partial y}\frac{\partial B_x}{\partial y}. \quad (36)$$

In the case considered ($\tilde{b}_z \equiv 0$, $\tilde{V}_z \equiv 0$) equations (28)–(29) reduce to the following equations:

$$b_x = \frac{i}{\omega}\Big(B_y\frac{\partial V_x}{\partial y} - V_y\frac{\partial B_x}{\partial y} - B_x\frac{\partial V_y}{\partial y}\Big), \qquad (37)$$

$$b_y = \frac{i}{\omega}\Big(ik_xB_xV_y - V_y\frac{\partial B_y}{\partial y} - ik_xB_yV_x\Big). \qquad (38)$$

We now compute the derivatives:

$$\frac{\partial b_x}{\partial y} = \frac{i}{\omega}\Big(\frac{\partial B_y}{\partial y}\frac{\partial V_x}{\partial y} + B_y\frac{\partial^2 V_x}{\partial y^2} -2\frac{\partial V_y}{\partial y}\frac{\partial B_x}{\partial y} - B_x\frac{\partial^2 V_y}{\partial y^2}\Big), \qquad (39)$$

$$\frac{\partial^2 b_x}{\partial y^2} = \frac{i}{\omega}\Big(2\frac{\partial B_y}{\partial y}\frac{\partial^2 V_x}{\partial y^2} + B_y\frac{\partial^3 V_x}{\partial y^3} -3\frac{\partial^2 V_y}{\partial y^2}\frac{\partial B_x}{\partial y} - B_x\frac{\partial^3 V_y}{\partial y^3}\Big), \qquad (40)$$

$$\frac{\partial b_y}{\partial y} = \frac{i}{\omega}\Big(ik_xB_x\frac{\partial V_y}{\partial y} + ik_xV_y\frac{\partial B_x}{\partial y} - \frac{\partial V_y}{\partial y}\frac{\partial B_y}{\partial y} -ik_xB_y\frac{\partial V_x}{\partial y} - ik_xV_x\frac{\partial B_y}{\partial y}\Big). \qquad (41)$$

By performing this differentiation we have taken full account of the weak nonhomogeneity of the coefficients in the y coordinate in the first order in the small parameter proportional to $1/R_\odot$. We further assume that perturbations are short- wavelength along the y coordinate and that the coefficients in the equation are independent of y, and seek a solution in the form $f \propto \exp\{ik_yy\}$, where $k_yR_\odot \gg 1$. We then obtain:

$$\frac{\partial b_x}{\partial y} = \frac{i}{\omega}\Big(ik_yV_x\frac{\partial B_y}{\partial y} - k_y^2B_yV_x -2ik_yV_y\frac{\partial B_x}{\partial y} + k_y^2B_xV_y\Big), \qquad (42)$$

$$\frac{\partial^2 b_x}{\partial y^2} = \frac{i}{\omega}\Big(- 2k_y^2V_x\frac{\partial B_y}{\partial y} - ik_y^3B_yV_x +3k_y^2V_y\frac{\partial B_x}{\partial y} + ik_y^3B_xV_y\Big), \qquad (43)$$

$$\frac{\partial b_y}{\partial y} = \frac{i}{\omega}\Big(- k_xk_yB_xV_y + ik_xV_y\frac{\partial B_x}{\partial y} -ik_yV_y\frac{\partial B_y}{\partial y} + k_xk_yB_yV_x - ik_xV_x\frac{\partial B_y}{\partial y}\Big). \qquad (44)$$

We now substitute the formulas derived into equations (35) and (36) and perform the substitution $B_x = B\sin\alpha$, $B_y = B\cos\alpha$ to obtain:

$$-(i\omega V_x + 2\Omega_zV_y)\frac{\partial \ln\rho}{\partial y} + k_y\omega V_x - 2ik_y\Omega_zV_y -2f_\Omega V_y = \frac{k_xk_y}{\rho}P - \frac{k_\perp^2k_y}{4\pi\rho\omega}B^2\sin\alpha\cos\alpha V_y + \frac{k_\perp^2k_y}{4\pi\rho\omega}B^2\cos^2\alpha V_x$$
$$+\left(\frac{i(k_\perp^2+k_y^2)}{4\pi\rho\omega}V_y\sin\alpha\cos\alpha - \frac{i(2k_\perp^2+k_y^2)}{8\pi\rho\omega}V_x\cos^2\alpha -\frac{ik_xk_y}{8\pi\rho\omega}V_y + \frac{ik_xk_y}{8\pi\rho\omega}V_x\sin\alpha\cos\alpha\right)\frac{\partial B^2}{\partial y}, \qquad (45)$$

$$-i\omega V_y + 2\Omega_zV_x = -\frac{ik_y}{\rho}P - -\frac{ik_\perp^2}{4\pi\rho\omega}B^2\sin^2\alpha V_y +\frac{ik_\perp^2}{4\pi\rho\omega}B^2\sin\alpha\cos\alpha V_x + \frac{1}{8\pi\rho\omega}$$
$$\times\left(2k_y\sin\alpha\cos\alpha V_x - 3k_y\sin^2\alpha V_y + k_x\sin\alpha\cos\alpha V_y\right)\times\frac{\partial B^2}{\partial y}. \qquad (46)$$

We now introduce the following designations :

$$U^2 = \frac{B^2}{4\pi\rho}, \qquad \mathcal{P} = \frac{P}{\rho}. \qquad (47)$$

We then supplement equations (45)–(46) by a third equation derived by eliminating the total derivative from equations (23)–(24), to obtain a set of three equations with variables V_x, V_y, and $\mathcal{P}$:

$$\left[k_y\omega - i\omega\frac{\partial \ln\rho}{\partial y} - \frac{k_\perp^2k_y}{4\pi\rho\omega}B^2\cos^2\alpha\right] - \left[\frac{i(2k_\perp^2+k_y^2)\cos^2\alpha - ik_xk_y\sin\alpha\cos\alpha}{8\pi\rho\omega}\frac{\partial B^2}{\partial y}\right]V_x$$
$$-\left[2ik_y\Omega_z + 2f_\Omega + 2\Omega_z\frac{\partial \ln\rho}{\partial y} - \frac{k_\perp^2k_y}{4\pi\rho\omega}B^2\sin\alpha\cos\alpha + \ +\frac{2i(k_\perp^2+k_y^2)\sin\alpha\cos\alpha - ik_xk_y}{8\pi\rho\omega}\frac{\partial B^2}{\partial y}\right]V_y$$
$$-k_xk_y\mathcal{P} = 0, \qquad (48)$$

$$\left[2\Omega_z - \frac{ik_\perp^2}{4\pi\rho\omega}B^2\sin\alpha\cos\alpha - \frac{k_y}{4\pi\rho\omega}\sin\alpha\cos\alpha\frac{\partial B^2}{\partial y}\right]V_x$$
$$-\left[i\omega - \frac{ik_\perp^2}{4\pi\rho\omega}B^2\sin^2\alpha + \frac{k_x\sin\alpha\cos\alpha - 3k_y\sin^2\alpha}{8\pi\rho\omega}\frac{\partial B^2}{\partial y}\right]V_y + ik_y\mathcal{P} = 0, \quad (49)$$

$$ik_xV_x + \left(ik_y + \frac{1}{\gamma}\frac{\partial\ln\rho}{\partial y}\right)V_y - \frac{i\omega}{c_s^2}\mathcal{P} = 0. \quad (50)$$

We then derive from the consistency condition of the equation set, i.e., from the condition that its determinant is equal to zero, our sought-for dispersion equation:

$$-\frac{1}{c_s^2}\left(k_y - i\frac{\partial\ln\rho}{\partial y}\right)\omega^4 + \left(k_\perp^2k_y - \frac{4i\Omega_zf_\Omega}{c_s^2} - ik_y^2\frac{\partial\ln\rho}{\partial y}\right.$$
$$+\left(k_y - i\frac{\partial\ln\rho}{\partial y}\right)\left(\frac{4\Omega_z^2}{c_s^2} - \frac{ik_y}{\gamma}\frac{\partial\ln\rho}{\partial y}\right)$$
$$+\frac{k_\perp^2k_y}{c_s^2}U^2 - \frac{ik_\perp^2}{c_s^2}U^2\sin^2\alpha\frac{\partial\ln\rho}{\partial y} +$$
$$+\frac{k_x\sin\alpha\cos\alpha - 3k_y\sin^2\alpha}{8\pi\rho c_s^2}\frac{\partial\ln\rho}{\partial y}\frac{\partial B^2}{\partial y}$$
$$\left.+\frac{i(2k_\perp^2 + k_y^2)\cos^2\alpha - 3ik_y^2\sin^2\alpha}{8\pi\rho c_s^2}\frac{\partial B^2}{\partial y}\right)\omega^2$$
$$+\left(2k_xk_y\Omega_z\left(\frac{\gamma-1}{\gamma}\frac{\partial\ln\rho}{\partial y} + \frac{f_\Omega}{\Omega_z}\right)\right.$$
$$-\frac{2k_\perp^2}{c_s^2}\left(f_\Omega + \Omega_z\frac{\partial\ln\rho}{\partial y}\right)U^2\sin\alpha\cos\alpha$$
$$+\frac{ik_y}{2\pi\rho c_s^2}\left(f_\Omega + \Omega_z\frac{\partial\ln\rho}{\partial y}\right)\sin\alpha\cos\alpha\frac{\partial B^2}{\partial y}$$
$$\left.+\Omega_z\frac{2k_\perp^2\sin\alpha\cos\alpha - k_xk_y}{4\pi\rho c_s^2}\frac{\partial B^2}{\partial y}\right)$$
$$\times\omega - k_\perp^2k_y(k_x\sin\alpha + k_y\cos\alpha)^2U^2$$
$$+\frac{ik_\perp^2k_y}{\gamma}(k_x\sin\alpha + k_y\cos\alpha)\cos\alpha U^2\frac{\partial\ln\rho}{\partial y}$$
$$+\frac{ik_\perp^2}{4\pi\rho c_s^2}(k_xk_y\sin^2\alpha - (2k_\perp^2 + k_y^2)\sin\alpha\cos\alpha)$$
$$\times\sin\alpha\cos\alpha U^2\frac{\partial B^2}{\partial y}$$
$$+\frac{ik_y}{8\pi\rho}(k_\perp^2k_x\sin\alpha\cos\alpha - k_x^2k_y - 4k_y^3\cos^2\alpha$$
$$+3k_y(k_x\sin\alpha + k_y\cos\alpha)^2 - 2k_\perp^2k_y\cos^2\alpha)\frac{\partial B^2}{\partial y}$$
$$+\frac{k_y}{8\pi\rho\gamma}(3k_xk_y\sin\alpha\cos\alpha$$
$$-(2k_\perp^2 + k_y^2)\cos^2\alpha)\frac{\partial\ln\rho}{\partial y}\frac{\partial B^2}{\partial y} = 0. \quad (51)$$

Dispersion equation (51) is extremely unwieldy and contains many parameters, which make it very difficult to analyze. At the same time, the particular case $B_y = 0$ is of considerable interest (note that the condition for the divergence of magnetic field allows the existence of such a solution not only in the local Cartesian, but even in the global spherical coordinate system). In this case $\sin\alpha = 1$, $\cos\alpha = 0$, all thermodynamical equilibrium parameters are homogeneous in y, and the dispersion equation acquires the following simple form:

$$\omega^4 - \left(k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2 - \frac{2i\Omega_zf_\Omega}{k_y}\right)\omega^2$$
$$-k_xf_\Omega c_s^2\omega + k_x^2k_\perp^2c_s^2U^2 = 0. \quad (52)$$

The second term in the coefficient at the squared frequency is evidently smaller than the first term in the case of solar parameter values. To estimate the third term, recall that the following condition is satisfied:

$$f_\Omega = \frac{2\Omega\cos\theta_0}{R_\odot}. \quad (53)$$

We then derive the following estimate for the third-to-second term ratio:

$$\frac{\frac{2i\Omega_zf_\Omega}{k_y}}{4\Omega_z^2} = \frac{\cos\theta_0}{\sin\theta_0}\frac{1}{k_yR_\odot} \simeq \frac{1}{k_yR_\odot} \ll 1. \quad (54)$$

Hence we can neglect the imaginary term in equation (52) by rewriting it in the following form:

$$\omega^4 - \left[k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2\right]\omega^2 - k_xf_\Omega c_s^2\omega$$
$$+k_x^2k_\perp^2c_s^2U^2 \simeq 0. \quad (55)$$

Note that the negligibly small imaginary term should be removed from equation (52), because imaginary terms in the dispersion equation for a dissipationless problem are unphysical.

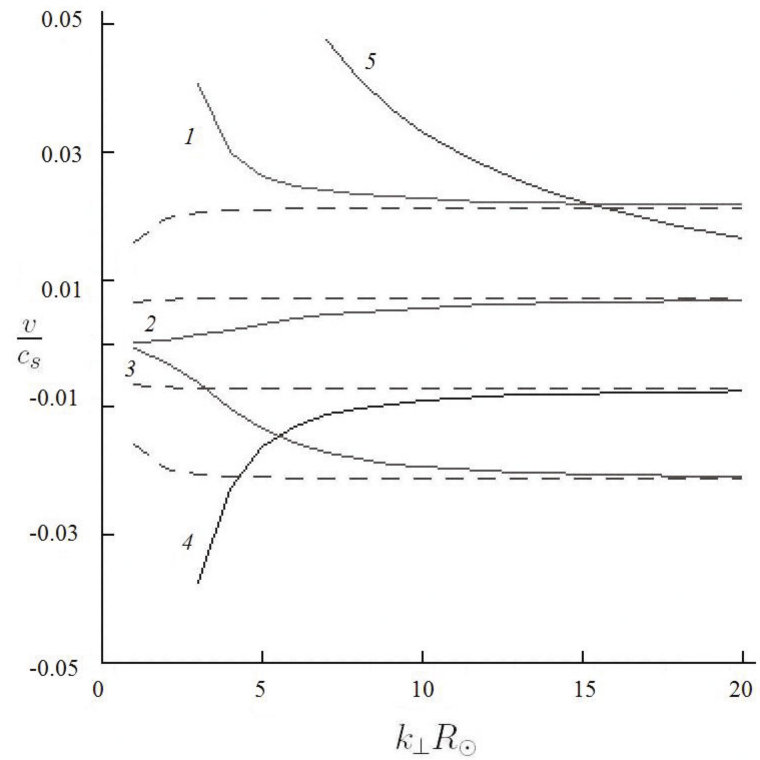

Fig. 1. Phase and group velocities of waves determined from equation (59). Here *1* shows the group velocity; *2*, the phase velocity for slow magneto-acoustic waves; *3*, the phase velocity for Rossby waves, and *4*, the group velocity. The dashed lines show the corresponding velocities in the absence of rotation ($\Omega \equiv 0$). Below the line *5* the domain is located where rotation effects on perturbations are important. $\varphi = 45°$, $\theta_0 = 45°$.

Equation (55) does not contain the term with frequency in the third power and therefore it allows, in principle, an analytical solution. However, such a solution proves to be extremely unwieldy and inconvenient to analyze. That is why in the general case we solve dispersion equation (55) numerically using Newton's iterative method. However, is has analytical solutions for particular cases (namely, the high- and low-frequency ones). We obtain the following approximate biquadratic equation for high-frequency modes and not too weak magnetic fields:

$$\omega^4 - \left[k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2\right]\omega^2 + k_x^2 k_\perp^2 c_s^2 U^2 \simeq 0. \quad (56)$$

Its solution has the following form:

$$\omega = \pm\left(\frac{k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2}{2} \times \left[1 \pm \sqrt{1 - \frac{4k_x^2 k_\perp^2 c_s^2 U^2}{[k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2]^2}}\right]\right)^{1/2}. \quad (57)$$

In the absence of rotation ($\Omega_z \equiv 0$) solution (57) provides the dispersion law for fast (the "+" sign in the second double sign) and slow ("−") magneto-acoustic waves, whereas in the absence of magnetic fields it provides the dispersion law for gravity-gyroscopic waves in the case of the "+" sign (in front of the round brackets) and degenerate ($\omega = 0$) entropic mode for the "−" sign.

However, of greatest interest for us are low-frequency solutions, which obey the following relation:

$$\left(k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2\right)\omega^2 + k_x f_\Omega c_s^2 \omega - k_x^2 k_\perp^2 c_s^2 U^2 \simeq 0. \quad (58)$$

Solutions of this equation are:

$$\omega \simeq -\left(k_x f_\Omega c_s^2 \pm\sqrt{k_x^2 f_\Omega^2 c_s^4 + 4k_x^2 k_\perp^2 c_s^2 U^2\left(k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2\right)}\right) /(2\left(k_\perp^2(U^2 + c_s^2) + 4\Omega_z^2\right)), \quad (59)$$

where the difference and sum in the numerator correspond to the dispersion law for slow magneto-acoustic and Rossby waves, respectively.

In the plots shown in Fig. 1 the derived analytical solution (59) is visually indistinguishable from the numerical solution and allows the group velocity to be directly computedь. To this end, let us use formula (53) and rewrite equation (59) in the following form:

$$\omega \simeq -\frac{k_x \frac{2\Omega_y}{R_\odot} c_s^2 \pm \sqrt{k_x^2 \left(\frac{2\Omega_y}{R_\odot}\right)^2 c_s^4 + 4k_\perp^2 k_x^2 c_s^2 U^2 \left(k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2\right)}}{2\left(k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2\right)}$$
$$= -k_\perp c_s \cos\varphi \frac{\frac{\Omega_y c_s}{R_\odot} \pm \sqrt{\frac{\Omega_y^2 c_s^2}{R_\odot^2} + k_\perp^4 U^2 (U^2 + c_s^2) + 4k_\perp^2 U^2 \Omega_z^2}}{k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2}. \quad (60)$$

We now use the definition of group velocity to derive:

$$\frac{d\omega}{dk_\perp} \simeq -\frac{c_s \cos\varphi}{k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2} \left(\frac{4\Omega_z^2 - k_\perp^2 (U^2 + c_s^2)}{4\Omega_z^2 + k_\perp^2 (U^2 + c_s^2)} \frac{\Omega_y c_s}{R_\odot} \times \sqrt{\frac{\Omega_y^2 c_s^2}{R_\odot^2} + k_\perp^4 U^2 (U^2 + c_s^2) + 4k_\perp^2 U^2 \Omega_z^2} \right.$$
$$\left. \pm \left[\frac{4\Omega_z^2 - k_\perp^2 (U^2 + c_s^2)}{4\Omega_z^2 + k_\perp^2 (U^2 + c_s^2)} \frac{\Omega_y^2 c_s^2}{R_\odot^2} + k_\perp^4 U^2 (U^2 + c_s^2) + 8k_\perp^2 U^2 \Omega_z^2 \right] \right). \quad (61)$$

We then use analytical formula (59) to derive the perturbed functions V_x, V_y, and $\mathcal{P}$ from equations (48)–(50). In view of the above assumptions, the equation set considered can be rewritten in the following, rather simple, form:

$$\omega V_x - \left[2i\Omega \sin\theta_0 + \frac{2\Omega \cos\theta_0}{R_\odot k_\perp \sin\varphi}\right] V_y$$
$$-k_\perp \cos\varphi \mathcal{P} = 0, \quad (62)$$

$$2\Omega \sin\theta_0 V_x - \left[i\omega - \frac{ik_\perp^2 U^2}{\omega}\right] V_y$$
$$+ik_\perp \sin\varphi \mathcal{P} = 0, \quad (63)$$

$$k_\perp \cos\varphi V_x + k_\perp \sin\varphi V_y - \frac{\omega}{c_s^2}\mathcal{P} = 0. \quad (64)$$

We derive from equation (62):

$$V_x = \frac{k_\perp \cos\varphi}{\omega}\mathcal{P}$$

$$+\frac{1}{\omega}\left[2i\Omega \sin\theta_0 + \frac{2\Omega \cos\theta_0}{R_\odot k_\perp \sin\varphi}\right] V_y. \quad (65)$$

We now introduce the following dimensionless quantities:

$$\tilde{z} = \frac{\omega}{k_\perp c_s}, \qquad W = \frac{\Omega}{k_\perp c_s}. \quad (66)$$

The above substitutions, combined with simple manipulations, yield the following formulae:

$$V_y = \frac{1}{c_s \tilde{z} \sin\varphi}$$
$$\times \frac{\tilde{z}^2 - \cos^2\varphi}{\frac{2W\cos\varphi}{\tilde{z}\sin\varphi}\left(i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right) + 1}\mathcal{P}, \quad (67)$$

$$V_x = \frac{1}{c_s \tilde{z}} \left(\cos\varphi + \frac{2W}{\tilde{z}\sin\varphi}\left[i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right] \frac{\tilde{z}^2 - \cos^2\varphi}{\frac{2W\cos\varphi}{\tilde{z}\sin\varphi}\left(i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right) + 1} \right) \mathcal{P}. \quad (68)$$

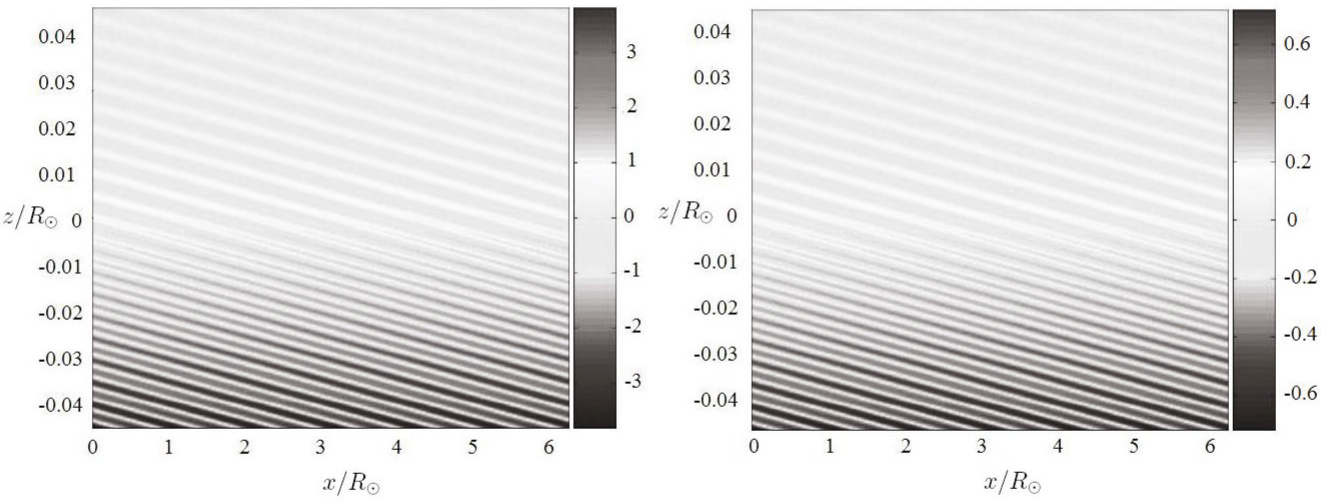

Fig. 2. Contours of the total (left) and thermodynamical (right) pressure in the vertical cross section for westward propagating Rossby waves.

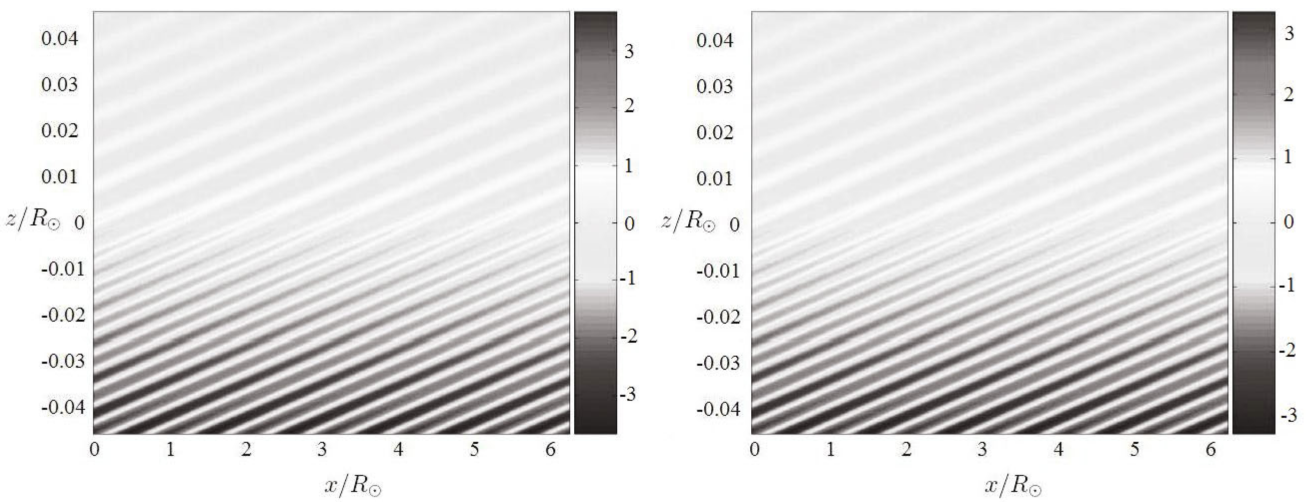

Fig. 3. Contours of the total (left) and thermodynamical (right) pressure in the vertical cross section for eastward propagating magneto-acoustic waves.

We now substitute the latter formulas into equations (37)–(38) to derive the formulas for the perturbed components of magnetic field:

$$b_x = \frac{k_y}{\omega} B V_y$$

$$= \frac{B}{c_s^2 \tilde{z}^2} \times \frac{\tilde{z}^2 - \cos^2\varphi}{\frac{2W\cos\varphi}{\tilde{z}\sin\varphi}\left(i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right) + 1}\mathcal{P}, \quad (69)$$

$$b_y = -\frac{k_x}{\omega} B V_y = -\frac{B\cos\varphi}{c_s^2 \tilde{z}^2 \sin\varphi}$$

$$\times \frac{\tilde{z}^2 - \cos^2\varphi}{\frac{2W\cos\varphi}{\tilde{z}\sin\varphi}\left(i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right) + 1}\mathcal{P}. \quad (70)$$

In the context of this paper of great importance is the fact that the vertical component of the wave vector is nonzero although $\tilde{b}_z \equiv 0$, $\tilde{V}_z \equiv 0$ in the case considered. This becomes immediately evident if we substitute formula (69) into equation (33), which describes the variation of the total pressure along the z coordinate. As a result, we have the following homo-

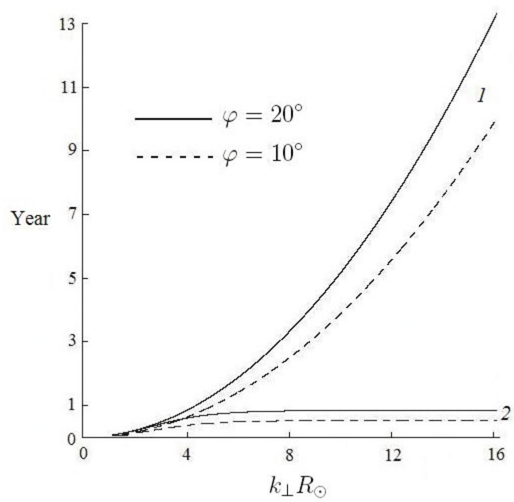

Fig. 4. Periods of beats 1 corresponding to carrier frequency 2. $\theta_0 = 45°$.

geneous ordinary differential equation:

$$\frac{\partial \tilde{P}}{\partial z} = -\frac{g}{c_s^2(1+F)}\tilde{P}, \tag{71}$$

where

$$F = \frac{U^2}{c_s^2 \tilde{z}^2} \times \frac{\tilde{z}^2 - \cos^2\varphi}{\frac{2W\cos\varphi}{\tilde{z}\sin\varphi}\left(i\sin\theta_0 + \frac{\cos\theta_0}{R_\odot k_\perp \sin\varphi}\right) + 1}. \tag{72}$$

The solution of equation (71) for the function of perturbed thermodynamical pressure is:

$$\tilde{P} = \hat{P}(0)\exp\left\{-\frac{g}{c_s^2(1+F)}z + ik_x x + ik_y y - i\omega t\right\}, \tag{73}$$

where $\hat{P}(0)$ is the amplitude of perturbed pressure at $z = 0$.

With further computations in mind, let us now estimate the g/c_s^2 ratio for the parameter values $R_\odot \simeq 6.9599(7) \times 10^8$ m, $c_s \lesssim 6$ km/s, $g \simeq 0.274$ km/s^2. We find:

$$\frac{R_\odot g}{c_s^2} \simeq 5.32(7) \times 10^3. \tag{74}$$

It is evident from formula (72) that F is a complex quantity. Hence the coefficient at z in the exponential function in formula (73) is also a complex quantity. Its imaginary part describes oscillations of perturbed pressure along the vertical coordinate and the real part, the decay of the amplitude of this pressure with height.

For our numerical computations and subsequent visualization of the results obtained we introduce yet another dimensionless quantity in addition to those introduced in equation (66):

$$A = \frac{U}{c_s}. \tag{75}$$

Dispersion equation (55) can then be rewritten in the following form:

$$\tilde{z}^4 - [4W^2\sin^2\theta_0 + 1 + A]\tilde{z}^2 - \frac{2W\cos\theta_0\cos\varphi}{k_\perp R_\odot}\tilde{z} + A^2\cos^2\varphi \simeq 0. \tag{76}$$

In the case of $A \gtrsim 1$, $k_\perp R_\odot \gg 1$ we obtain the following solutions:

$$\tilde{z} \simeq \frac{1}{\sqrt{2}} \pm \Big[1 + A + 4W^2\sin^2\theta_0 \pm\sqrt{(1 + A + 4W^2\sin^2\theta_0)^2 - A^2\cos^2\varphi}\Big]^{1/2}, \tag{77}$$

which represent fast (“+” in the second double sign) and slow (in the case of “−”) magnetohydrodynamical gravity-gyroscopic waves.

In the case of $A \ll 1$, $|\tilde{z}| \ll 1$, slow magnetohydrodynamical gravity-gyroscopic waves are subject to the Rossby effect:

$$\tilde{z} \simeq \frac{-\frac{W\cos\theta_0\cos\varphi}{k_\perp R_\odot} \pm \sqrt{\frac{W^2\cos^2\theta_0\cos^2\varphi}{k_\perp^2 R_\odot^2} + A^2\cos^2\varphi[4W^2\sin^2\theta_0 + 1 + A]}}{4W^2\sin^2\theta_0 + 1 + A}. \tag{78}$$

In this case the following condition is satisfied for W:

$$W = \frac{\Omega}{k_\perp c_s} = \frac{M}{m}\cos\theta_0\cos\varphi, \tag{79}$$

where for solar values:

$$M \simeq \sqrt{0.11}, \qquad m = 6. \tag{80}$$

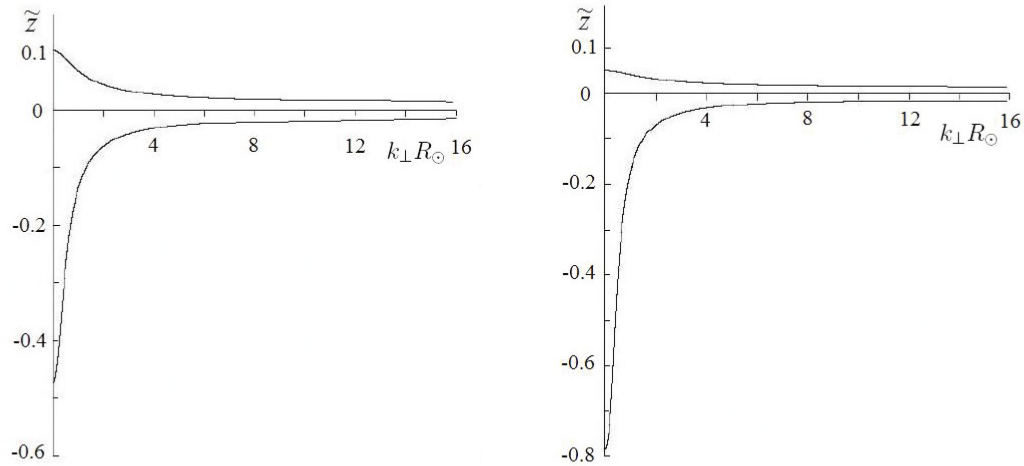

Fig. 5. Dependence of dimensionless phase velocity on dimensionless wavenumber for the cases $\varphi = 10°$ (left) and $\varphi = 20°$ (right), $\alpha = \theta_0 = 45°$.

5. RESULTS AND DISCUSSION

Figure 1 shows the dispersion curves for the phase velocities of slow magneto-acoustic waves derived in accordance with formula (59). The dashed lines show the corresponding curves in the case of no rotation. It is evident that the westward propagating slow magneto-acoustic wave exhibits a well-defined Rossby effect. The same figure shows the group velocities corresponding to these phase velocities and computed by formula (61). It is interesting that if the phase of the wave propagates westward then the packet of such waves drifts eastward and vice versa.

Rotation affects substantially the dynamics of wave structures if the Kibel–Rossby number obeys the following constraint:

$$R_o = \frac{v_{gr}}{2\Omega_z l_\perp} \ll 1, \qquad (81)$$

where v_{gr} is the group velocity of the wave and $l_\perp$, its horizontal scale length. This results in the following inequality:

$$\frac{v_{gr}}{c_s} \ll 4\pi \frac{\Omega \sin\theta_0}{k_\perp c_s} = 4\pi W. \qquad (82)$$

In Fig. 1 we mark the line corresponding to condition (82) by number *5*.

Figures 2 and 3 show the contours of the perturbations of the thermal and total (magnetic pressure included) pressure at the surface of the vertical cross section for both branches of slow magneto-acoustic waves discussed in this paper. We believe that the formation of tilted low-pressure ducts shown in Figs. 2 and 3 should facilitate the emergence of matter from lower levels along these ducts. In this case such emergence should be more efficient when it is driven by a westward propagating Rossby wave (low-pressure ducts produced by Rossby waves propagate in the direction opposite to that of the local velocity of solar rotation making it likely that the matter from subphotospheric layers would be "swept up" more efficiently in these ducts).

We believe the fact that dispersion law (59) is unambiguously indicative of the presence of beats in the system studied to be a result of great importance. Let us consider, for the sake of simplicity and clarity, the $z = 0$ layer, equal-amplitude perturbations with frequencies ω_1 and ω_2, and separate their real parts in formula (73) (because, evidently, only real parts have any physical meaning), and derive the following formula for the superposition of such perturbations:

$$\tilde{P} = \hat{P}(0)\cos(k_x x + k_y y - \omega_1 t) + \hat{P}(0)\cos(k_x x + k_y y - \omega_2 t) = \hat{P}(0) \times \cos\left(\frac{\omega_2 - \omega_1}{2} t\right) \cos\left(k_x x + k_y y - \frac{\omega_1 + \omega_2}{2} t\right). \qquad (83)$$

Given that, according to formula (59), ω_1 and ω_2 are close in absolute value, but have opposite signs, we have oscillations at the carrier frequency of $(\omega_2 - \omega_1)/2$ modulated by the frequency $(\omega_1 + \omega_2)/2$ at every fixed point of the horizontal plane considered.

Note that such beats cannot, in principle, be found either in the magnetohydrodynamical formulation of the problem without the allowance for rotation—in this case we have for slow magneto-acoustic waves $|\omega_1| \equiv |\omega_2|$ and, consequently, $\omega_1 + \omega_2 \equiv 0$, or in the formulation of the problem with rotation, but without magnetic field, because in this case one of the entropic branches ($\omega_2 \equiv 0$) is degenerate (it corresponds to the sign "−" in the asymptotic formula (59)).

We now use formula (59) to explicitly determine the carrier and beat frequencies:

$$\frac{\omega_2 - \omega_1}{2} = \frac{\sqrt{k_x^2 f_\Omega^2 c_s^4 + 4k_x^2 k_\perp^2 c_s^2 U^2 \left(k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2\right)}}{k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2}, \quad (84)$$

$$\frac{\omega_1 + \omega_2}{2} = -\frac{k_x f_\Omega c_s^2}{k_\perp^2 (U^2 + c_s^2) + 4\Omega_z^2}. \quad (85)$$

It follows from formula (59) that the beat frequency for sufficiently weak magnetic fields ($U^2 \ll c_s^2$) depends only very slightly on the field magnitude and practically coincides with the Rossby frequency for the purely magnetohydrodynamical case.

Figure 4 shows the dependence of the periods determined using formulas (84)–(85) on $k_\perp R_\odot$ for the case of typical solar parameter values.

In addition, we also numerically analyze the general dispersion equation (51) and show the results in Fig. 5. It is immediately apparent from this figure that in the long-wavelength domain in the cases $\varphi = 10°, \varphi = 20°$ the asymmetry in the phase velocity of the wave may result in beats.

6. CONCLUSIONS

We now list the main conclusions of this paper.

(1) Strong magnetic fields suppress wave effects associated with rotation.

(2) In the case of observed “solar” values of magnetic-field intensity ranging from several G to several kG the Rossby effect shows up in the branch of slow magneto-acoustic waves propagating against the direction of local solar rotation.

(3) Dispersion law allows the existence of Rossby waves having only horizontal components of perturbed velocity vectors and magnetic field; the wave vector has nonzero vertical component in this case, i.e., perturbations are baroclinic and not barotropic.

(4) Contours of minimum perturbed pressure are tilted with respect to the horizontal plane and this the matter from lower layers with higher pressure and magnetic-field pressure to emerge along these "ducts".

(5) The superposition of eastward propagating slow magneto-acoustic waves and westward propagating Rossby waves results in long-period beats that show up against short-periodic oscillations at the carrier frequency. We believe that such beats may contribute to the dredge-up of magnetized plasma from deep layers onto the solar surface and affect the development of solar activity with periods from 9 to 13 years.

We consider the main result of our work to be that beats of westward propagating MHD Rossby waves and magneto-acoustic waves propagating in the opposite direction may explain why active regions arranged with a large spatial period (at active longitudes) are located at low and mid-latitudes and, mostly, along the same latitude.

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REFERENCES

1. Y.Q. Lou, Astrophys. J. Suppl. **540**, 1102 (2000).
2. C.G. Rossby, J. Mar. Res. **2**, 38 (1939).
3. M.V. Nezlin, E.N. Snezhkin, *Vikhri Rossby i spiral'nye struktury: Astrofizika i fizika plazmy v opytakh na melkoi vode* (Rossby vortices and spiral structures: Astrophysics and plasma physics in shallow-water experiments) (Nauka, Moscow, 1990), p. 240 [in Russian]
4. Pedloski, J., *Geofizicheskaya gidrodinamika*, (Mir, Moscow, 1984), v. 1, p. 398 (Russian translation of Pedloski, J., *Geophyscial Fluid Dynamics*, (Springer, New York, 1979), v. 1).
5. A. Gill, *Atmosphere-Ocean Dynamics*, (Academic Press, San Diego, 1982).
6. L.D. Landau, E.M. Livshits, *Teoriya polya*, (Field theory) (Nauka, Moscow, 1988), p. 512 [in Russian].
7. V.V. Mustsevoi, A.A. Solov'ev, Astron. Zh. **74**, 254 (1997).
8. L.D. Landau, E.M. Livshits, *Elektrodinamika sploshnykh sred*, (Electrodynamics of continua) (Nauka, Moscow, 1988), p. 620 [in Russian].