

# DISSIPATION INSTABILITIES IN THE ACCRETION DISK

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**Abstract.** The model of a geometrically thin gaseous disk in the external gravitational potential is considered. The dynamics of small nonaxisymmetric perturbations in the plane of the accretion disk with dissipative effects is investigated. It is showed, that conditions of development and parameters of unstable oscillation modes in the optically thick accretion disk are strongly depended on the models of viscosity and opacity.

The possibility of the development of various types of instabilities in a thin gaseous disks is very attractive one for understanding different aspects of the accretion-disk (AD) phenomenon. In order to explain the required large values of dissipative coefficients, the concept of the turbulent viscosity has been used, which may be caused by the developed turbulence of a gaseous medium arising from the loss of stability. On the other hand, the nonlinear evolution of unstable oscillation modes may be responsible for many nonstationary phenomena in accreting systems.

There are four unstable oscillation modes in the framework of the standard  $\alpha$ -model of an accretion disk [1]. Two of them are acoustic [6-11], one is viscous and the other one is thermal [2-11]. A distinctive feature of the dynamic viscosity  $\eta = \sigma\nu$  in the model of the accretion disks is it's dependence on the surfase density  $\sigma$  and on the disk half-thickness  $h$  ( $\nu$  is the kinematic viscosity). The perturbation of the dynamic viscosity  $\tilde{\eta}$  is responsible for the formation of all unstable four modes. However, in all the above-cited papers on the dynamic of linear perturbation is suggested that viscosity simultaneously changes with changing of accretion-disk parameters.

In our present work, we extend our research to time delay of the viscosity influence on the thermal, viscous and acoustic instabilities. In the construction of a different viscous models AD is suggested that viscosity caused by the developed turbulence  $\eta \sim \sigma u_t \ell_t$ , where  $u_t$  and  $\ell_t$  is the characteristic velocity of the large-scale turbulent pulsations and it's characteristic size, respectively [12]. The fundamental energy is contained in the large-scale pulsations, however dissipation of the energy is the case in the small-scale pulsations. By this means, as the local conditions changes in the accretion disk, there are two factors, which involves the change delay of the turbulent viscosity. Firstly, since the turbulence may be caused by the nonlinear evolution of unstable oscillation modes, that for formation of the developed turbulence is required of the characteristic time  $\tau_1$ . The first approximation may be thought of as  $\tau_1$  is proportional to the build-up time of instabilities. Secondly, there are delay  $\tau_2$ , associated with transfer of an energy from large-scale pulsations to small-scale pulsations. Hence, as accretion-disk parametrs changes (of the temperature and density), value of the actual viscosity time delay from instant value of the dynamic viscosity  $\eta_*$  by characteristic time  $\tau = \tau_1 + \tau_2$ . For standard  $\alpha$ -model AD  $\eta_* \sim \alpha\sigma\Omega h^2$ , where  $\Omega$  is the Keplerian angular velocity. The first approximation law of relaxation viscosity  $\eta$  to value  $\eta_*$  may be written in the following form:

$$\frac{d\eta}{dt} = \frac{\eta_* - \eta}{\tau}, \quad (1)$$

We restrict ourselves to the case of small perturbation with  $kr \gg 1$  and  $m/r \ll k$  ( $k$  is the radial wavenumber and  $m$  is azimuthal wavenumber), which allows us to use

WKB approximation and seek the solution in the form

$$\tilde{f} = f_1 \exp(-i\omega t + ikr + im\varphi), \quad (2)$$

where  $\omega$  is the complex frequency of a mode. Equilibrium quantities are denoted by the subscript "0".

### The influence of the delay of the viscosity.

In the general case  $\tau > 0$  we obtain the five-order dispersion relation. This is equation describes five oscillatory modes. Four of them were considered previously at  $\tau = 0$  [11]. The inclusion of the delay  $\tau > 0$  gives rise to new oscillatory mode is the second-viscous mode, beside  $\text{Re}(\omega) = 0$  and  $\text{Im}(\omega) < 0$  at any values of another parameters. Rewriting equation (1) in terms of the result (2) we obtain in the linear approximation :

$$\frac{\eta_1}{\eta_0} = \frac{1}{1 - i\omega\tau} \frac{\eta_{*1}}{\eta_0}, \quad (3)$$

where  $\frac{\eta_{*1}}{\eta_0} = \frac{\sigma_1}{\sigma_0} + \frac{\nu_{*1}}{\nu_0} = (1 + \delta_\sigma) \frac{\sigma_1}{\sigma_0} + \delta_h \frac{h_1}{h_0}$ ,  $\delta_\sigma = \left( \frac{d \ln \nu_*}{d \ln \sigma} \right)_0$ ,  $\delta_h = \left( \frac{d \ln \nu_*}{d \ln h} \right)_0$  [11].

For a standard  $\alpha$ -model accretion disk in the radiation-pressure-dominated region the increment of all four unstable modes decreases, with the increase of characteristic time of the delay  $\tau$ , as indicated by fig. 1. Stabilization of the thermal and viscous oscillation modes ( $\text{Im}(\omega) \lesssim 0$ ) occurs at  $\tau \gtrsim 100/\Omega$ , while the acoustic mode tend to become stable at  $\tau \simeq 1/\Omega$ .

### The influence of the opacity

The conditions of development for unstable oscillation modes very strongly depend on the model of opacity. We assume that opacity  $\bar{\kappa}$  is a function of  $\sigma$  and  $h$  (in other words, of density and temperature). The linear approximation yields

$$\frac{\bar{\kappa}_1}{\bar{\kappa}_0} = \Delta_\sigma \frac{\sigma_1}{\sigma_0} + \Delta_h \frac{h_1}{h_0}, \quad \left\{ \Delta_\sigma = \left( \frac{d \ln \bar{\kappa}}{d \ln \sigma} \right)_0, \quad \Delta_h = \left( \frac{d \ln \bar{\kappa}}{d \ln h} \right)_0 \right\}. \quad (4)$$

In the case of the Thomson scattering  $\bar{\kappa} = \bar{\kappa}_{es} = 0, 4\text{sm}^2/\text{g}$  ( $\Delta_\sigma = 0$ ,  $\Delta_h = 0$ ), and for Kramers law  $\bar{\kappa} = \bar{\kappa}_{ff} \propto \rho T^{-7/2}$  ( $\Delta_\sigma = 1$ ,  $\Delta_h = -8$ ). For low temperature protoplanetary disks  $\bar{\kappa} \propto T^2$  [13] ( $\Delta_\sigma = 0$ ,  $\Delta_h = 4$ ). In the "hot" limit of Foulkner model [14]  $\bar{\kappa} \propto \rho T^{-5/2}$  ( $\Delta_\sigma = \frac{2 + \beta_0}{2(1 + 3\beta_0)}$ ,  $\Delta_h = -\frac{12 + \beta_0}{2(1 + 3\beta_0)}$ ), but in the "cold" limit  $\bar{\kappa} \propto \rho^{1/3} T^{10}$  ( $\Delta_\sigma = 1/3$ ,  $\Delta_h = 59/3$ ). Here  $\beta_0 = P_{0rad}/(P_{0rad} + P_{0gas})$ ,  $P_{gas}$  is the gas pressure,  $P_{rad}$  is the radiation pressure.

As indicated by fig. 2 the thermal mode of the oscillation become unstable even in the case  $\beta_0 = 0$  at  $\Delta_\sigma < 0$  and  $\Delta_h > 0$ . Stabilization of the acoustic-mode occurs at a high negative value  $\Delta_\sigma$ . However, in the framework of standard model AD [1] acoustic modes of the oscillations prove to be unstable both in the inner radiation-dominated region ( $P_{rad} \gg P_{gas}$ ,  $\bar{\kappa} = \bar{\kappa}_{es}$ ), and in the external gaseous region ( $P_{gas} \gg P_{rad}$ ,  $\bar{\kappa} = \bar{\kappa}_{ff}$ ).

## The influence of the viscosity

As the second (elastic) viscosity  $\mu_0$  increases, the increment of the sound waves decreases until the imaginary part vanishes at  $\mu_0 = \mu_{0\text{crit}}$ . The perturbation decay ( $\text{Im}(\omega) < 0$ ) when  $\mu_0 > \mu_{0\text{crit}}$ . As shown in the work [11] the quantity  $\mu_{0\text{crit}}$  only weakly depends on  $\beta_0$ , so that  $\mu_{0\text{crit}} = 2 \div 3\nu_0$ .

The increments and conditions for instability development very strongly depend on the parameters  $\delta_\sigma$  and  $\delta_h$ . By now there are many models of turbulent viscosity with different values  $\delta_\sigma$ ,  $\delta_h$ . For standard  $\alpha$ -model  $W_{r\varphi} = -\alpha p$  [1] and  $\delta_\sigma = 0$ ,  $\delta_h = 2$ . For modifications of  $\alpha$ -models  $W_{r\varphi} = -\alpha(p_g/p)^{N/2}p$  ( $N = \text{const}$ ) [2,4,5] and  $\delta_\sigma = \frac{N\beta_0}{2(1+3\beta_0)}$ ;  $\delta_h = \frac{4+\beta_0(12-7N)}{2(1+3\beta_0)}$ .

The functions  $\text{Im}[\omega(\delta_\sigma)]$  and  $\text{Im}[\omega(\delta_h)]$  are in qualitative agreement with  $\text{Im}[\omega(\Delta_\sigma)]$  and  $\text{Im}[\omega(\Delta_h)]$  accordingly. In the terms of the mentioned models of turbulent viscosity, both acoustic modes of the oscillations are unstable at  $\mu_0 < \mu_{0\text{crit}}$ , because for all these models  $\delta_\sigma > 0$  and  $\delta_h > 0$ . The acoustic oscillations stabilize at large negative values of  $\delta_\sigma$ .

## Conclusion

A linear analysis has been performed to examine the radial-azimuthal instability of accretion disks. The main results are as follows:

First, the existence of unstable modes is associated completely with the perturbation of dynamic viscosity  $\eta = \sigma\nu$  and, consequently, it is determined by the dependence of  $\nu(\sigma, h)$ . If we set  $\tilde{\eta} \equiv 0$ , all four oscillatory modes (including the thermal and viscous modes in the radiation-dominated limit) would decay with decrement  $\text{Im}(\hat{\omega}) \sim -\nu_0 k^2$ .

Second, the azimuthal perturbation wavenumber  $m$  in terms of the WKB approximation for a short-wave perturbations with  $kr \gg 1$  not affected by the increments of unstable modes and that is responsible for Dopplershift of the frequency  $\hat{\omega} = \omega - m\Omega$ .

Third, in the radiation-pressure-dominated accretion disk the thermal and viscous unstable modes stabilize at large values of the characteristic time scale of the delay  $\tau$ , while the acoustic mode tend to become stable at small values  $\tau$ .

Finally, the conditions of development and parameters of the thermal, viscous and acoustic instabilities in the geometrically thin and optically thick accretion disk are strongly depended on the models of viscosity and opacity. In the gas-pressure-dominated accretion disk the thermal mode tend to become unstable with decreases of the values  $\delta_\sigma, \Delta_\sigma$  and with increases of the values  $\delta_h, \Delta_h$ . The acoustic mode at any gas-to-radiation pressure ratio tend to become stable with decreases of the values  $\delta_\sigma, \Delta_\sigma$ . An additional point to emphasize is that the sound waves stabilize at large values of the second (elastic) viscosity.

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### References

1. Shakura N.I. & Sunyaev R.A., 1973, A&A 24, 337
2. Lightman A.P. & Eardley D.M., 1974, ApJ 187, L1
3. Shakura N.I. & Sunyaev R.A., 1976, MNRAS 175, 613
4. Taam R.E. & Lin D.N.C., 1984, ApJ 287, 761
5. Szuszkiewicz E., 1990, MNRAS 244, 377
6. Wallinder F.H., 1990, A&A 237, 270
7. Wallinder F.H., 1991, MNRAS 253, 184
8. Wallinder F.H., 1991, A&A 249, 107
9. Okuda T. & Mineshige S., 1991, MNRAS 249, 684
10. Wu X-B. & Yang L-T., 1994, ApJ 432, 672
11. Khoperskov A.V. & Khrapov S.S., 1995, Astronomy Letters 21, N3, 347
12. Landau L.D. & Lifshits E.M., 1986, *Gidrodynamicheskii kurs*, W: Science
13. Lin D.N.C., 1981, ApJ 246, 972
14. Foulkner J., Lin D.N.C. & Papaloizou J., 1983, MNRAS 205, 359

### Figure Captions

Fig. 1. Dependence of the imaginary part of the frequency in terms of the angular velocity  $\text{Im}(\omega)/\Omega$  on the without dimensional wavenumber  $kh_0$  a),b) and on the without dimensional time of delay  $\tau\Omega$  c),d).

Fig. 2. Dependence of the imaginary part of the frequency in terms of the angular velocity  $\text{Im}(\omega)/\Omega$  of acoustic, thermal and viscous oscillation modes on the value parameter  $\Delta_\sigma$  a),b) and on the value parameter  $\Delta_h$  c),d) at  $kh_0 = 1$ . The solid and short dashed lines represent  $\beta_0 = 0$  and  $\beta_0 = 1$ , respectively.